Demand, Structural Interdependence and Economic Provisioning

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Introduction

The function of an economic system is to enable the individuals who comprise it to meet their material needs. Different sorts of systems do this in different ways; some do the job "better" than others, according to various criteria (the rate of growth of per capita income; access to subsistence; distributive justness; and so forth). Owing to the division of labor, industrial and post-industrial economies are characterized by a high degree of structural interdependence: technical interdependence among productive sectors, interdependence between demand and employment, interdependence between state and economy. Structural interdependence has been the subject of economic analysis since the emergence of classical political economy in the 17th century, most notably in the *Tableau Economique*, in Marx's schemes of reproduction, in the post-Marx writings on the trade cycle, in Input-Output economics & activity analysis, and in the Sraffa model.

This paper will focus on the role of demand in the context of such models of structural interdependence. Once an economy reaches a stage of economic development in which the technology enables a substantial portion of the population to enjoy a standard of living significantly above subsistence, "the material needs" of an economic community themselves become difficult to define, because they are interconnected with the relations of production in complex ways. In particular, demand comes to play a key role in the subsequent development of the system. Wages are no longer analogous to the fuel that is needed to power an engine, or the fodder than a team of oxen need to enable them to pull a plow. Aggregate demand drives growth, and the composition of demand regulates the allocation of resources. Under modern capitalism, the situation is complicated by the fact that the composition of demand is shaped in large part by the marketing activity and wage policies of powerful oligopolistic entities. The paper will reflect on these issues in the light of the class of structural models associated with Leontief, Pasinetti, Lowe and Sraffa. These models have the merit of avoiding the pitfalls of the conventional treatment of demand in terms of price elastic demand functions, but they have made only tentative progress in explaining the evolution of demand. The aim of the paper is not to provide a full-fledged theory of demand, but to assess how these structural models have treated demand and suggest how the theory of demand can be further developed within the broad framework of such models.

Modeling the Provisioning Process

The first steps taken by the mercantilists in the 16th and 17th centuries were necessarily primitive, and with few exceptions (Mun) barely scratched the surface of the problem. Focused as they were on commerce—on flows of goods and money—the mercantilists had little to say about how those goods get produced, allocated and reproduced in such a way as to enable the economy to persist through time; on development and structural change they offered nothing at all. In the 17th century William Petty, in a series of works that were not published in his lifetime, correctly identified production as the ultimate source of income, an insight that represented a significant advance on the misleading mercantilist view that trade is the basis of prosperity. Petty also introduced the crucial idea that an economy is prosperous in so far as it is capable of producing a surplus over and above the wage goods and material inputs consumed in the production process. By the next century, even before the onset of industrialization in Britain, political economists had thoroughly internalized Petty's outlook.

Bernard Mandeville, less a political philosopher or social scientist than a mischievous wag who specialized in poking the eye of bourgeois complacency, recognized not only that production is what enables a society to thrive, but also that once the economy has advanced beyond a bare subsistence standard of living, production is to a large extent driven by demand. The metaphorical hive of Mandeville's *Fable of the Bees* (1724) is a sophisticated economic system, exhibiting a fairly extensive division of labor, that until its conversion to virtuous austerity is an engine of self-reproduction. In its thriving phase, the hive undergoes no structural evolution, though Mandeville allows for changes in fashion.

To Petty's notion of surplus the physiocrats added an explicit recognition of the interconnectedness of production. The *Tableau Economique* depicts the economy as a network of interconnected sectors and social classes. Economic activity is conceived as a circular process of production and consumption in which the outputs of the agricultural and manufacturing sectors serve as inputs into each others' production processes. Adam Smith made two significant advances on the rudimentary physiocratic model. First, he recognized that the manufacturing sector is as capable as agriculture of generating a surplus, and consequently plays an indispensable role in growth and development. . Second, he sketched out the mechanism by which a market economy coordinates atomistic self-interested behavior to enable the material reproduction of the system. It is a remarkable feature of market economies that commodities are produced not in random quantities, but in amounts that roughly coincide with what can be sold; that resources get channeled out of sectors whose products are wanted in smaller quantities than before, and into sectors whose products are now in higher demand and indeed might not even have been imagined just a few years earlier; and that incomes are generated which enable the members of the system to purchase the goods that its productive activity has created. The system is, in other words, able to reproduce itself. The mechanism which brings about this coordination is of course the intersectoral movement of capital in pursuit of its highest return, a process which, as described by Smith, manifests itself through the gravitation of market prices of goods toward long-period cost of production. None of this is meant to suggest that the mechanism unfolds seamlessly, or can be relied upon to generate optimal outcomes. The point is simply is that this mechanism must play a part in any account of how a capitalist system provisions itself and how it evolves through history.

David Ricardo clarified and refined Smith's argument. Ricardo's friend Thomas Robert Malthus raised prescient questions about the ability of a market economy to sustain aggregate demand at levels sufficient to prevent the system's stagnation or decline. Marx developed the classical surplus approach further, drawing directly upon the *Tableau Economique* to construct his Volume II reproduction schemes, which expose the roles that technical change, structural imbalances and monetary phenomena may play in triggering crises.

The first few generations of neoclassical economists were no less concerned than the classicals and Marx with provisioning and economic structure. Alfred Marshall's definition of economics as man in the ordinary business of life reflects his concern with the problem of material provisioning, and the attention he paid to the institutional framework is evidence of his recognition that the structural features of the economy are crucial to the provisioning process. While Jevons and Walras directed their attention mainly to what we would call technical theoretical problems, they did so with a view to providing a tool to address real-world problems. The Austrian School's distinction between lower and higher order goods points to structural interconnections that underpin the provisioning process. Hayek's production triangles, Böhm-

Bawerk's concept of the roundaboutness of production and Lachmann's emphasis on the complementarities among different capital goods as a factor in cyclical fluctuations—all these are indications that the Austrians were and are duly aware of the structural features of economic activity. Welfare economics and the socialist calculation debates are, if nothing else, a great hashing-out of the criteria by which to evaluate how effectively different institutional frameworks meet people's provisioning requirements.

Structural and Behavioral Models

Though economists have routinely taken account structural factors and provisioning, or did so at least until the late decades of the last century, it is nevertheless useful to distinguish between structural and behavioral models (Nell, 1984; Lowe, 1964).Behavioral models are principally concerned with how market participants with given characteristics react to stimuli provided by their economic environment; the objective is to predict the pattern of responses triggered by a change in circumstances. Equilibrium is defined as a set of mutually consistent decentralized choices; economic theories are serviceable generalizations about how these choices are made. The derivation of such generalizations requires a number of pre-analytical conjectures about the basic characteristics and motives of agents. Typically it is supposed that individuals and firms are optimizers—i.e. they are driven by the desire to maximize or minimize something, like utility or profits or costs—and that they are rational in that their actions are designed to bring them closer to their respective optima. By virtue of these assumptions, the behavioral approach can conveniently be translated into the precise mathematics of calculus, linear programming or axiomatic set theory. There are numerous questions for which behavioral considerations are of undeniable importance, questions for example relating to the impact of different economic policies. But behavioral models ignore a whole class of issues which are crucial to understanding how economic systems function. All economic activity takes place within a definite social setting comprised of institutions, laws, rights, social obligations, behavioral conventions and so forth. Behavioral models take this social context for granted, without enquiring into the conditions necessary for its perpetuation.

The maintenance and reproduction of the social matrix is the sine qua none for the systematic operation of any stimulus response mechanism; that is precisely why economists often take it for granted. But agents must somehow learn the rules imposed upon them by the existing mode of production, that is to say, they must learn what sorts of behavior will enable them to survive and flourish in a given social environment. They are constrained not just by resource availability, but also by legal and moral prescriptions and by the cognitive blinders that any social framework imposes upon the human mind. The most fundamental problem from an economic standpoint concerns the conditions required for the material continuation of society. If, in the long run, these conditions are not met, the social context which is today taken for granted will tomorrow be the subject of history.

Structural models address those basic issues to which I have just alluded. They focus upon the requirements for physical reproduction of the socio-economic system rather than upon agents' behavioral responses to a particular set of conditions. Instead of treating the social context as a fait accompli, structural models attempt to discover what patterns of behavior are compatible with the existence and continuity of the institutions, class relations and production processes which characterize the economic system. The objectives are, first, to identify and explain the rules which preserve the system's viability, and then to determine the implications these rules have for the system's evolution over time.

The models of mainstream economics are behavioral. Demand and supply functions summarize the reactions of economic actors to market stimuli in a given institutional context, and equilibria are characterized by the balancing of forces which direct the decisions of buyers and sellers. Classical models are structural. In the simplest constructions they determine the prices reintegrate the economy's production processes and enable the system to reproduce itself. At higher levels of complexity, classical analysis attempts to show how capitalist institutions, class behavior and production relations reinforce one another; it investigates the patterns of accumulation implicit in capitalist production; and it inquires into the relations connecting consumption, distribution and accumulation.

A Classical-Keynesian Model

The classical theorists and Marx wanted to understand how a market economy generates and distributes a social surplus. Their interest in this question derived from two premises: that in a capitalist economy the surplus manifests itself as profits and rents; and that it is the social class that receives profits which undertakes the accumulation of capital. In classical surplus theories the process by which relative prices are determined does not simultaneously determine distribution and quantities; these are analyzed separately. The data of the classical theory are: (i) the size and composition of the social product; (ii) the technical conditions of production; and (iii) some distribution parameter—either the real wage or the profit rate. If we take the gross output vector as given and assume that all capital is circulating capital, that one method of production is available for the production of each commodity, that wages are not advanced, that there is no joint production and that all commodities are basic in the sense of Sraffa[1], then relative prices and the profit rate are determined by the solution to the following system:

$$\mathbf{p} = \mathbf{pA}(1+r) + w\mathbf{l}$$
$$(1) \qquad p_1 = 1$$
$$w = w^*,$$

where $\mathbf{p}$ is a row vector of n prices; $\mathbf{A} = [a_{ij}]$ is an n X n matrix each element of which represents the amount of commodity i required to produce a unit of commodity j; $\mathbf{l}$ is a row vector of labour input requirements; r is the profit rate; and $w = w^*$ is the real wage, measured in terms of the numeraire (commodity 1), which we shall regard as parametric.

Starting from fundamental data about the size and composition of the social product, the technical conditions of production and the real wage, this system determines the n-1 relative prices and the real wage. Manipulation of (1) yields the solution:

$$(2) \qquad \mathbf{p} = w\mathbf{l}[\mathbf{I}-\mathbf{A}(1+r)]^{-1},$$

where $\mathbf{I}$ is the identity matrix. Post-multiplication of (2) by the column vector $\mathbf{e}_1 = [1, 0, ..., 0]$ gives $\mathbf{pe}_1 = w\mathbf{l}[\mathbf{I}-\mathbf{A}(1+r)]^{-1}\mathbf{e}_1$; but since $p_1 = 1$ this reduces to:

$$(3) \qquad 1 = \mathrm{w}\mathbf{l}[\mathbf{I}-\mathbf{A}(1+r)]^{-1}\mathbf{e}_1.$$

An increase in r will always cause the scalar $\mathbf{l}[\mathbf{I}-\mathbf{A}(1+r)]^{-1}\mathbf{e}_1$ to increase; since the vector components of that scalar are fixed, the equality sign in (3) can be maintained only by a decline in

[1]A basic commodity is one that enters directly or indirectly into the production of every commodity in the system.

w. This is true regardless of which commodity or composite of commodities is chosen as numeraire; thus there will always be a monotonic inverse relationship between *w* and *r*. The trade-off has the form of a polynomial expression, and its precise shape will normally depend on the dimensions of **A**; on the technical conditions of production, that is, on the magnitudes of the elements of **A** and **l**; and on the choice of numeraire; in general, the relationship will not be linear.[2]

The separate treatment of pricing, distribution and output sharply differentiates the surplus approach from marginalist theory, where a grand unifying principle, factor substitution, permits (in fact, requires) the determination of all economic variables at a single stroke. Surplus theories utilize less intricate lines of causality, but, by dividing the analysis into distinct logical stages, are able to bring within the scope of economics issues which marginalist theory tends to ignore—e.g., questions relating to the distribution of property and wealth (as opposed to income), to the formation of tastes and preferences, or to the determinants of technical change. Where marginalist theory explains little by aspiring, in its formal analytics, to do too much, the surplus approach places less ambitious demands on its theoretical core, and is therefore able to deal usefully with a broader range of problems.

Implicit in the classical conception of a market economy is a set of quantity relations that will be satisfied in long-period equilibrium. Let the elements of the column vector $\mathbf{q} = [q_1, ..., q_n]$ represent the gross outputs of commodities 1 through *n*. The elements of a second column vector $\mathbf{y} = [y_1, ..., y_n]$ are the net outputs or final demands for the various commodities. A vector **q** must be adequate to replace the commodities used up in production and to satisfy the final demand. In the long-period, the economy will not produce more of any commodity than is demanded (demand for additions to inventories is included in **y**), so we have the equations:

[2]In orthodox theory the function of the price mechanism is to clear markets by bringing quantities supplied into equality with quantities demanded. Prices perform a somewhat different task in the classical theory. The production of any commodity requires inputs from the various other sectors of the economy. During a round of production each industry produces at least enough output to replace what is used up by itself and by other sectors. But at the end of each production period all of the output of any commodity is in the possession of the industry that produced it; before another round of production can begin, the economy must somehow manage to transfer a portion of each output from the industry that produced it to the industries that use it as an input; this is accomplished by means of the price system. Competition causes the price vector will assume a configuration that permits the reproduction of the economy with a uniform profit rate. In both the classical and marginalist theories, prices play an indispensable coordinating role in the allocation of resources; but the nature of this role differs in the two cases. In the marginalist theory prices exist to enable the economy to accommodate the problem of scarcity, while classical prices of production make possible the swaps necessary for the economy to reproduce itself.

(4) $$\mathbf{Aq}+\mathbf{y} = \mathbf{q}, \text{ with solution}$$

(5) $$\mathbf{q} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{y}.$$

Unless the net output vector **y** (or some other n-component vector of gross and net outputs) is taken as parametric, the quantity system will be underdetermined (by *n* degrees of freedom) and no solution will be possible.

The quantity relations (4) simply reflect the condition that the quantity demanded of each commodity must match the amount of it produced. This equality of supply and demand refers only to produced goods and not, of course, to labor. The uniform profit rate condition and the implicit assumption that capitalists seek the highest possible rate of return imply that no entrepreneur will tend to produce more or less of a good than he can sell; but the same uniformity condition does not require that the supply and demand for labour be equalized. It is essential to note that while the equality of supply and demand for commodities is a necessary feature of long-period equilibrium, the forces of supply and demand do not determine long-period natural prices; nor do they explain outputs, since the vector of final demands is parametric.

It can easily be shown that given **y** and r, the value of net output is equal to the sum of wages and profits distributed within the economy. From the price equations, $\mathbf{pA}+r\mathbf{pA}+w\mathbf{l} = \mathbf{p}$, or

(6) $$\mathbf{p}(\mathbf{I}-\mathbf{A}) = w\mathbf{l}+\mathrm{r}\mathbf{pA}.$$

Post-multiplication of (6) by **q** gives $\mathbf{p}(\mathbf{I}-\mathbf{A})\mathbf{q} = w\mathbf{lq}+\mathrm{r}\mathbf{pAq}$. Thus, we have $\mathbf{py} = w\mathbf{lq}+r\mathbf{pAq}$, or $\mathbf{py} = W+\Pi$, where W is the economy's wage bill and Π is the amount of profits paid to the owners of capital.

The earliest attempt to disaggregate Keynes's model and express it in the form of a Leontief-type matrix was undertaken by Richard Goodwin (1949), who was writing too early to have had as an objective the integration of the models of Keynes and Sraffa; but his paper is a brilliant performance and deserves more attention that it has received, especially from Post-Keynesians. Several attempts have been made to construct formal models that integrate the classical and Keynesian theories (Pasinetti 1974; Eatwell 1979; Kurz, 1985). The model presented here (which I have utilized elsewhere, see Mongiovi 1991, 1992) draws upon these contributions and upon a less well known paper by Miyazawa &

Masegi (1963), who set up their model in terms of value transactions; so we will need to introduce some modifications to derive a model whose parameters and variables refer to physical quantities. Once this is done we will not have far to go to obtain a model of output determination which is fully compatible with the classical theory and which permits the investigation of problems that cannot easily be handled by earlier formulations.

Let us begin by considering an economy in which constant returns prevail. We will also suppose that all goods are basics, that only one technique is available for the production of any commodity, and that all capital is circulating capital. As usual **A** is a square matrix of unit input coefficients; **q** and **y** are column vectors of gross outputs and final demands (or net outputs) respectively. Once produced, any final good can be used either to satisfy consumption demand or to expand the economy's stock of plant and equipment; this fact can be taken into account if the vector of final demands is expressed as the sum of two subvectors:

(7) $$\mathbf{y} = \mathbf{y}^c + \mathbf{y}^I.$$

Each element y_i^c (i = 1, ... , n) of the column vector $\mathbf{y}^c$ represents the amount of commodity i demanded for consumption purposes; each element y_i^I of the vector $\mathbf{y}^I$ indicates how much of commodity i the economy wishes to channel into the expansion of productive capacity.[3]

Assume that there exist two non-overlapping social classes — workers, who receive wages, and capitalist-entrepreneurs, who receive profits. Each of these classes is presumed to allocate a certain proportion of its money income to the consumption of each of the commodities produced by the economy. If we associate the subscripts w and π with the working class and capitalist class respectively, we can define the consumption coefficient c_{iw} (or $c_{i\pi}$) as the number of additional units of commodity i demanded by the economy with each additional unit of wage (or profit) income; thus, $p_i c_{iw}$ (or $p_i c_{i\pi}$) is the marginal propensity of the wage-earning (or profit-receiving) class to spend its income on good i. For convenience, we suppose that prices and the wage rate are measured in units of an accounting money, which we shall call dollars.

[3] It is clear that the elements of $\mathbf{y}^c$ and $\mathbf{y}^I$ must be non-negative. In many instances, though, we can expect that various elements of the two vectors will be zero. For example, the demand for machine tools for consumption purposes is bound to be zero. All goods are basics, but they may not enter *directly* into the investment demand vector. Thus, corn may be a basic commodity because it enters into the production of gasohol; yet there may be no direct investment demand for it. Thus the entry for corn in the consumption demand vector will be positive, but the corresponding entry in the investment demand vector will be zero.

The consumption coefficients can be arranged into a matrix

$$\mathbf{C} = \begin{bmatrix} c_{1w} & c_{1\pi} \\ c_{2w} & c_{2\pi} \\ \vdots & \vdots \end{bmatrix}$$

which is subject to the following constraints: first, it must be true that $\mathbf{pC} \leq [1, 1]$; that is, the marginal propensity to consume of each class must be less than or equal to one, and the MPC of at least one class must be strictly less than one. Second, we impose the simplifying conditions

$$p_i c_{iw} = \gamma_{iw}$$

$$p_i c_{i\pi} = \gamma_{i\pi},$$

($i = 1, ..., n$); these conditions, which express the assumption that agents always spend the same proportion of their incomes on each good, are not essential, and nothing of substance would be changed by dropping them.

A change in incomes or relative prices will almost always entail changes in **C;** these changes are apt to be somewhat complicated and, owing to the fact that they are largely grounded in subjective impulses, will be somewhat unpredictable in magnitude and perhaps direction. The second set of conditions on **C** is therefore nothing more than a convenient way of managing the complexity of consumption behavior; if we are prepared to make detailed assumptions, based for example on Engel curve data, about how the elements of **C** respond to income and price changes, we can substitute another set of conditions for the simple ones given here.

It will be useful to define a matrix **V** whose elements represent the values added by labour and by produced means of production to the price of each commodity:

$$\mathbf{V} = \begin{bmatrix} wl_1 & wl_2 & \cdots & wl_n \\ r\sum p_i a_{i1} & r\sum p_i a_{i2} & \cdots & r\sum p_i a_{in} \end{bmatrix},$$

where w, r and p_i ($i = 1, ..., n$) represent as before the real wage, the profit rate and the prices of commodities. The profit rate is taken as a datum; and since production is, by assumption, characterized by constant returns, w and relative prices can also be considered given and fixed once a numeraire is chosen. A noteworthy feature of this matrix is the dependence of each of its elements upon the prior solution of the economy's price equations. The income of each class is given by

$$\mathbf{Vq} = \begin{bmatrix} W \\ \Pi \end{bmatrix},$$

where W and Π are the money incomes of the working and capitalist classes. From this it follows that

(8) $$\mathbf{y}^{c} = \mathbf{CVq}.$$

We know that in equilibrium the gross output vector must satisfy the quantity relations (4) $\mathbf{q} = \mathbf{Aq} + \mathbf{y}$. Substituting (7) and (8) into (4) gives: $\mathbf{q} = \mathbf{Aq} + \mathbf{CVq} + \mathbf{y}^{I}$. Manipulating this last expression, we obtain:

(9) $$\mathbf{q} = [\mathbf{I} - \mathbf{A} - \mathbf{CV}]^{-1}\mathbf{y}^{I}.$$

The inverse matrix in (9) can be manipulated further, giving the result:

(10) $$\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1}[\mathbf{I} - \mathbf{CV}(\mathbf{I} - \mathbf{A})^{-1}]^{-1}\mathbf{y}^{I}.$$

Thus, given **A**, **C**, and the profit rate (which determines **V**), any vector of investment demand will determine a unique vector of gross outputs.

The first term—$(\mathbf{I} - \mathbf{A})^{-1}$—in the expression which multiplies $\mathbf{y}^{I}$ in (6.4) is the familiar Leontief inverse. The interpretation of the expression $[\mathbf{I} – \mathbf{CV}(\mathbf{I} - \mathbf{A})^{-1}]^{-1}$ is less obvious; but it can be shown, with not too much effort, to be a perfect matrix analogue to the simple Keynes-Kalecki multiplier.

Let Q and Y represent the *values* of gross and net output for the economy as a whole; the ratio of the value of inputs to the value of gross output is given by a, so that $Y/Q = (1–a)$. The coefficients c_{w} and c_{π} are workers' and capitalists' propensities to consume; W/Y and Π/Y are the shares of wages and profits in national income. Finally, define $v_{w} = W/Q$ and $v_{\pi} = \Pi/Q$ as the additions to the value of gross output attributable to wages and profits when Q increases by one dollar. The simple Keynesian multiplier is written as $1/(1–c)$. But if workers and capitalists have different propensities to consume, c is actually a weighted average of their different propensities to consume, where the weights are the shares of wages and profits in national income. Thus we have:

(11) $$c = c_{\mathrm{w}}(W/Y) - c_{\pi}(\Pi/Y).$$

Kalecki's multiplier (Kalecki 1969, ch.5) in this way takes account of differences in saving ratios between classes, and so may be written:

$$\frac{1}{1-[c_w v_w + c_\pi v_\pi)/(1-a)]},$$

Multiplying and dividing (6.5) by Y/Q and then substituting into (6.6) gives:

$$\frac{1}{1-[c_w(W/Y)-c_\pi(\Pi/Y)]}$$

which inspection reveals to be perfectly symmetric with the matrix formulation $[\mathbf{I} - \mathbf{CV}(\mathbf{I}–\mathbf{A})^{-1}]^{-1}$ (Miyazawa & Masegi 1963, pp. 90–91).

By combining the Leontief and Keynes-Kalecki multipliers, system (10) places in sharper focus the transmission mechanism by which autonomous changes in demand are translated into still larger changes in output. An autonomous increase in any of the elements of $\mathbf{y}^{\mathrm{I}}$ will (through the operation of the Leontief inverse) lead to an increased demand for produced inputs. If the necessary labour is available, there will be an increase in the incomes of newly-hired workers and of the owners of newly-hired capital in those input sectors. Once this increase in incomes occurs, the Keynes-Kalecki multiplier becomes operative as the receivers of the just-created income begin to spend it on final goods. There follows an increase in the demand for inputs required to produce *those* final commodities. From this point on, the Leontief and Keynesian multipliers operate together until the initial increase in spending wears itself out in the usual way.

The net output vector is easily derived. Pre-multiplying both sides of (6.4) by $(\mathbf{I} - \mathbf{A})$, we obtain $(\mathbf{I} - \mathbf{A})\mathbf{q} = [\mathbf{I} - \mathbf{CV}(\mathbf{I}–\mathbf{A})^{-1}]^{-1}$. But since $(\mathbf{I} - \mathbf{A})\mathbf{q} = \mathbf{q} - \mathbf{Aq} = \mathbf{y}$, we have:

(12) $$\mathbf{y} = [\mathbf{I} - \mathbf{CV}(\mathbf{I}-\mathbf{A})^{-1}]^{-1}\mathbf{y}^{I}.$$

The values of the gross and net output vectors can of course be obtained by pre-multiplying (6.4) and (6.7) by **p,** which (under conditions of constant returns) may be taken as fixed once the profit rate is given:

$$Q = \mathbf{pq} = \mathbf{p}(\mathbf{I}-\mathbf{A})^{-1}[\mathbf{I} - \mathbf{CV}(\mathbf{I}-\mathbf{A})^{-1}]^{-1}\mathbf{y}^{I}$$
$$Y = \mathbf{py} = \mathbf{p}[\mathbf{I} - \mathbf{CV}(\mathbf{I}-\mathbf{A})^{-1}]^{-1}\mathbf{y}^{I}$$

It is an easy matter to demonstrate that the value of net output, Y, is equal to the sum of wages and profits generated by the economy. We begin by manipulating the price equations to obtain $\mathbf{p}(\mathbf{I}-\mathbf{A}) = w\boldsymbol{l}+r\mathbf{pA}$. Post-multiplication by **q** gives:

(13) $$\mathbf{p}(\mathbf{I}-\mathbf{A})\mathbf{q} = w\boldsymbol{l}\mathbf{q} + r\mathbf{pAq}.$$

But $\mathbf{p}(\mathbf{I}-\mathbf{A})\mathbf{q} = \mathbf{y}$; so we have $\mathbf{py}=w\boldsymbol{l}\mathbf{q}+r\mathbf{pAq}$, or $Y=W+\Pi$, where W and Π are, as before, the wages and profits paid out by the economy in the course of producing **q**.

It can also be shown that in equilibrium the value of net output must be equal to the sum of planned consumption and investment expenditure. From the solution for the net output vector (12) we have:

$$[\mathbf{I} - \mathbf{CV}(\mathbf{I}-\mathbf{A})^{-1}]\mathbf{y} = \mathbf{y}^{I}$$
$$\mathbf{y}-\mathbf{CVq} = \mathbf{y}^{I}$$
$$\mathbf{y} = \mathbf{y}^{I}+\mathbf{CVq}$$

Premultiplying by the price vector gives $\mathbf{py}=\mathbf{py}^{I}+\mathbf{pCVq}$, or $Y=I+C$, where I and C are planned investment and consumption expenditures.

A few observations ought to be made here regarding the operation of the multiplier in this model. It is clear, first of all, that a change in investment need only occur in a single sector to set off a chain reaction throughout the economy. In the end we will find that the components of the consumption vector will not be what they were, and that the outputs of all basic commodities will have changed.[4] An advantage of the matrix formulation is its ability to trace the complex consequences of an initial change, or set of changes, in parameters.

A second aspect of the model that is of interest is its explicit consideration of the influence exerted by the distribution of income (represented by the value-added matrix **V**) on the solution. Investment demand gives rise to a secondary consumption demand; since it is presumed that workers and capitalists have different consumption patterns, this secondary demand will vary according to how income is distributed between wages and profits. The secondary consumption demand will in turn determine what commodities will be required as inputs in the next round of production, and the input coefficients of these required means of production will determine, through **V**, how much the incomes of the working class and the capitalist class will change in that production period. Tertiary changes in the composition of consumer demand will naturally ensue, and so the process must continue until the multiplier runs its course. Moreover, owing to the layered nature of the production process, in which commodities are produced by commodities in a complicated circular network, a change—even a small one—in the profit rate or real wage can radically alter the composition of gross output. Any change in distribution will activate a complex sequence of adjustments the results of which cannot be known *a priori*. An increase, for example, in the profit rate may cause a particular q_i to either rise or fall; and the change in q_i can be large or small regardless of the size of the initial change in the profit rate. The result depends entirely upon the characteristics of the parameter **A, l** and **C**.

Finally, there is associated with any gross output vector **q** a level of employment $N = \boldsymbol{l}\mathbf{q}$. An implication of the matrix multiplier is that a change in the composition of demand can alter the level of employment, even if total expenditure remains constant; indeed, GDP and employment are as likely to move in opposite directions as in the same direction; (Kurz 1985, pp. 130–132, also makes this point). More importantly, there is no reason to suppose that the level of employment determined by the solution vector **q** will utilize all of the labor available at the going wage; no

[4] If only one component of $\mathbf{y}^I$ changes, or if all changes in its components are in the same direction, then the outputs of basic commodities will all change in the same direction — increasing if the initial change was positive, decreasing if the initial change was negative. If, on the other hand, some elements of $\mathbf{y}^I$ increase while others decrease, the impacts on the outputs of basic commodities cannot be predicted *a priori*. They may all move together, or some may rise while others fall, depending upon input requirements, the magnitudes of the initial changes in the investment demand vector , and the effects of the resulting changes in incomes on the pattern of consumption.

forces are at work which can ensure that the economy will gravitate toward a position of full employment.

Consider a situation in which a substantial amount of unemployment is present. Clearly a reduction in the wage with no change in investment will not provide a remedy. Even if the decline in w is accompanied by an increase in the profit rate, the MPC of capitalists is smaller than that of workers, so that there will be a net decline in consumption demand, and consequently a net decline in gross output and employment. We have here a vindication of Keynes' opposition to wage reductions as a response to unemployment.[5] (In the unlikely event that capitalists have a higher MPC than workers, a decline in wages will indeed increase employment, though not necessarily to the extent required.) What may be required is a *higher* wage rate, but even this is no guarantee; in any case, during periods of high unemployment, the bargaining position of labour is weak, so that it is difficult to conceive of market forces effecting an increase in real wages.

Dynamic Aspects of the Matrix Multiplier

We turn briefly to the dynamic aspects of the matrix multiplier derived in the preceding section. The model deals with situations that are fully adjusted with respect to the price vector and therefore does not exposes the sequential effects of a change in the exogenous distribution variable on outputs and employment. These sequential adjustments occur, by definition, outside of equilibrium, that is, in a sphere within which our analysis does not penetrate. It is supposed here that such adjustments do not exhibit the same regularity that can be attributed to the forces that determine the solution to system (10), and therefore are not susceptible, without the adoption of special assumptions, to an exact analysis. The adoption of this supposition does not imply that questions concerning the movements of the variables through time are of no interest. On the contrary, Goodwin (1949) has shown that these movements can have important consequences for the cyclical behavior of the economy. He has investigated the dynamic aspects of a matrix

[5] There is a possibility that the decline in wages will push the economy closer to full employment. If the commodities which enter into the consumption bundles of capitalists are produced by sufficiently labor intensive techniques, or require sufficiently large quantities of produced inputs, the increased demand for these goods created by the rise in r may more than compensate for the loss in employment due to a wage reduction. However, to the extent that (i) profits represent a smaller portion of national income than wages, (ii) the consumption expenditures of the capitalist class as a whole are smaller than those of the working class, and (iii) capitalists and workers consume the same commodities, this possibility will not be of great importance.

multiplier by introducing a set of income-expenditure lags, the presence of which creates the possibility of macroeconomic oscillations.

More problematic, perhaps, is the question of the meaning of the notion of "long-period equilibrium" in a Keynesian context, in which investment represents changes in productive capacity, while long-period equilibrium has traditionally been regarded as a static position with productive capacity fully adjusted to aggregate demand. In *The General Theory* Keynes (1936, p.47–48) describes the long-period level of employment as the level consistent with the existing state of long-term expectations. The model outlined here can be interpreted in an analogous way. The given vector $\mathbf{y}^I$ expresses the long-term expectations of entrepreneurs as well as the other factors (objective and subjective) that influence investment at a particular historical moment. The solution of system (10) determines the gross outputs that will be generated by the demand for these investment goods. The analysis does not require that the investment vector has the property of persistence or quasi-permanence. Given $\mathbf{y}^I$, we can identify the level of employment toward which the economy will gravitate. If it is legitimate to suppose (as we have done here) that the elements of $\mathbf{y}^I$ change slowly—that is, that the recursive effects of the gravitation process on investment are of negligible magnitude—then the static character of the model will not be problematic.[6]

A Framework for Modeling Investment

If aggregate output is demand-constrained rather than supply-constrained, a useful investment theory will have to be compatible with the theory of effective demand. Here a difficulty arises. The classical writers and Marx saw the production of a physical surplus as a precondition for economic expansion. Growth takes place when part of an existing net product is ploughed back into the production process, so that output can be enlarged in the next period. But in order for the surplus to be ploughed back, a decision must be made to refrain from consuming it; the saving decision must be taken prior to any investment decision which means that saving must be treated

[6]The solution to system (10) can be conceived as stationary state. But without an explicit theory of investment there is no reason to suppose that the elements of $\mathbf{y}^I$ would remain constant outside of particular historical circumstances—that is, outside of a situation of extremely limited duration. Naturally if the elements of y^I change rapidly with the passing of time, the static formulation would be useful mainly as an illustrative device to put in clearer focus the effects on production and employment of a change in investment. But it would be possible to modify the model to take account of dynamic considerations. We shall not attempt here to enlarge the analysis in this direction; however it is worth noting that this dynamic project might be grounded in the work of Goodwin (1949) and Pasinetti (1981).

as *analytically* prior to investment.[7] This conflicts with the Keynesian understanding of how market economies operate. The classical view of accumulation appears to require that causality run from saving to investment; and to ensure that this causal relation could be made consistent with the macroeconomic equilibrium condition that leakages from the expenditure flow must be matched by injections into it, the classicals, in the absence of a better alternative, had to fall back on a second unkeynesian device, Say's Law.

Marx assigned investment a crucial role in the competitive struggle for markets among different capitals. This aspect of the accumulation process suggests that there is a component to investment that is independent of the rate of return on capital. If a firm's survival depends upon its ability to innovate and expand, it will engage in a certain amount of investment regardless of what its rate of return happens to be. The degree to which any particular firm will be inclined (or compelled) to engage in such autonomous investment will depend upon its absolute size (measured, for example, by its productive capacity or by the average size of its labor force), its size relative to its competitors, and the degree of concentration of the industry in which it operates. It is probable that autonomous investment will be positively related to the degree of concentration, which is generally taken to reflect the degree of monopoly power. Where an industry is dominated by a small number of large firms, competition is apt to be far more intense and the penalties for laziness more severe (Clifton 1977, Schumpeter 1950).

If the broad competitive market structure of the economy is taken as given, we can conceive of a set of parameters α_{ij} ($i,j = 1,..., n$) which represent the amount of autonomous investment demand by sector j for commodity i. The magnitude of each such parameter will depend upon the competitive characteristics of sector j, with the α_{ij}s presumably increasing with the degree of concentration, and upon a variety of nonquantifiable considerations such as product characteristics and the "corporate culture" which characterizes the managements of the firms in that sector. This formulation is problematic in the sense that specification of the market structure presumes some notion of the scale of output in various sectors. We may get around the difficulty by supposing that agents take decisions against a background of particular historical circumstances,

[7] The origins of this idea can be traced at least as far back as the Physiocratic literature. It should be noted that no attempt is here being made to deny that economic growth requires both the production of a surplus and the application of part of that surplus to accumulation. Rather, what will be argued is that it is demand which calls the surplus into existence in the first place and so which provides the ultimate impetus to growth.

and that these circumstances establish broad levels of productive activity within and across sectors. This does not mean that we take productive capacity as fixed, but that managers take decisions in the context of an economy in which, e.g., automobile production is on the order of 12 million units per year divided among three firms, rather than 300 thousand units per year divided among fifty firms. We define $\alpha_i = \Sigma\alpha_{ij}$ and $\boldsymbol{\alpha} = [\alpha_i]$; this vector represents the autonomous component of $\mathbf{y}^I$, that is, the component of investment demand that is not dependent on the rate of return on capital.

The next issue that needs to be considered is whether the rate or the mass of profits should be used to determine investment. To use the mass of profits would be inappropriate for two reasons. First, given physical input requirements and the profit rate, the level of profits will vary with the level of productive activity. Taking the mass of profits as a datum presupposes that the output vector is known from the start; but since the purpose of the exercise is ultimately to explain the output vector. Second, the demand for investment goods will depend not upon the total amount of profits present in the economy, but upon how these profits are apportioned among sectors. While aggregate investment spending might be highly correlated with aggregate profits (Eisner 1963), we are concerned with the determination of a vector of sectoral outputs; this means that the vector of induced investment demands cannot be treated as a simple function of Π, the sum of the economy's profits, but must depend upon the vector of sectoral profit levels, (Π_1, Π_2, ... Π_n). This approach again runs into the difficulty that, Π_i depends upon sectoral output q_i.[8]

The profit rate also has an advantage over absolute profits in terms of motivational rationale. The rate of return on capital is an index of the benefits that can be expected to accrue from investment; thus the greater is the profit rate the greater will be the incentive for capitalists to take the trouble and run the risks associated with capacity expansion. Absolute profits cannot serve as well in this regard, since they are specified without reference to the value of the capital upon which they are earned; profits may be high, but if the value of the capital required to generate those profits is high as well, the rate of return on investment will be relatively low.

Let y_{ij} represent the amount of investment demand for commodity *i* induced by profitability in sector *j*; $y_{ij} = y_{ij}(r_j)$, where r_j is the rate of return on capital invested in sector *j*. Now let us define $y_i = y_i(r_1, ..., r_n)$ as the total amount of induced investment demand for commodity *i*. Finally, we

[8]Kalecki (1969) discusses investment in a dynamic setting, and is therefore able to take a lagged change in profits as the main independent variable; but Kalecki's aims are different from ours in that the outputs determined along his time path are not long-period positions.

may write $\mathbf{y}_r^I = \mathbf{y}(r_1, ..., r_n)$ for the vector of investment demand related to profitability. Under competitive conditions, the economy will gravitate toward a uniform profit rate; it is precisely through investment responses to differential profit rates, here modeled explicitly, that a uniform profit rate is established. We may therefore write $\mathbf{y}_r^I = y(r)$, where r is the exogenously determined normal rate of profit. Thus we have $\mathbf{y}^I = \boldsymbol{\alpha} + \mathbf{y}(r)$. Combining this result with our matrix multiplier in system (10) gives:

$$\text{(14)} \qquad \mathbf{q} = (\mathbf{I}-\mathbf{A})^{-1}[\mathbf{I}-\mathbf{CV}(\mathbf{I}-\mathbf{A})^{-1}]^{-1}[\boldsymbol{\alpha} + \mathbf{y}(r)].$$

When the profit rate and money wage are specified, the price equations (1) and system (6.3) allow us to obtain a determinate solution for prices, the wage rate, investment, sectoral outputs and employment.

The construction summarized in system (7.3) captures a number of key elements in the classical approach to investment, while avoiding the trap posed by Say's Law. The classical insight that a physical surplus must be present for accumulation to take place is retained; but in an obviously Keynesian fashion, the surplus is itself called into existence by demand, as represented by the vector $[\boldsymbol{\alpha} + \mathbf{y}(r)]$. Saving is determined by investment through the income generation process, rather than the other way round.

But the limitations of this formalization are also evident. For a start, the precise relationship between investment demand and the profit rate expressed by the equations $\mathbf{y}_r^I = \mathbf{y}(r)$, is a behavioural relation that is particularly apt to be tenuous and inexact. There are no *a priori* reasons to expect the components of $\mathbf{y}_r^I$ to exhibit any substantial elasticity with respect to changes in the general rate of profit. The law of competition implies that intersectoral profit rate *differentials* will induce systematic adjustments in the investment vector; but this does not explain why there should be a significant correlation between investment demand and the general profit rate established by competitive market forces.[9] A positive correlation between investment and the profit rate, it will be recalled, has been supported on two grounds. First, all other things being equal, a higher profit

[9]The same point holds when long-period equilibrium is characterized by intersectoral profit rate differentials due to institutional impediments to the free mobility of capital: monopolistic forces, barriers to entry, government regulations, etc. Once the long-period position is established, the rationale for a systematic behavioral relationship leading from the vector of sectoral profit rates to the vector of investment demands lose much of its force.

rate entails a larger surplus available for expansion of the capital stock; second, the profit rate represents an incentive for capitalists to enlarge the stock of assets from which they derive their incomes. But since in our model it is demand (one component of which is investment) which calls forth the surplus, the latter cannot, *in the long-run*, constitute a constraint upon accumulation; therefore the profit rate cannot be regarded as an index of the economy's capacity to accumulate. And, while the profit rate clearly provides an incentive to investment when rates of return are not equalized, its ability to perform that function once competition has eliminated intersectoral differentials is by no means evident.

Furthermore, even if it were possible to defend the existence of a systematic positive relationship between the investment vector and the profit rate, the exact form of the relationship will depend upon expectations. Since we are mainly concerned with the long-period, we need consider only long-term expectations; short-period expectations, which are particularly volatile, can be ignored. If the state of long-term expectations can be taken as fixed, it is possible at least to conceptualize a relationship of the form $\mathbf{y}_r^I = \mathbf{y}(r)$. But once that relationship is put to its intended use, in comparative static analysis, serious difficulties arise. It is inconceivable that long-term expectations will remain unchanged in the face of an autonomous change in the profit rate. Even a small change in expectations may radically alter the form of the relationship between $\mathbf{y}_r^I$ and r; at the same time, there is no way to know with any degree of certainty how changes in the profit rate will influence expectations. Thus we are deprived of the possibility of constructing a model of investment behavior which is both rigorous and simple.

Conclusion

Investment is a complex phenomenon that cannot be reduced to a set of neat mathematical formulae. This was recognized by the classicals, who, in their theoretical discussions, took care not to impose an artificial simplicity on economic behavior. How then is investment to be explained?

It might be argued that investment must in the long-run conform to long-period expectations about the growth of demand. Once the latter are given, the components of the investment vector will fall into place. But this is less an explanation than a way to avoid the difficult questions altogether.

Investment depends upon a wide range of factors. Current profitability, particularly if it is high or low by historical standards, may exert some influence, as the classicals supposed. The availability and cost of financing (to which the level and structure of interest rates are relevant) will almost certainly also have a bearing on investment spending. But the connection between the conditions of financing and the level of desired investment is far more complex, and far less direct, than conventional theoretical and policy discussions suggest. While the logical basis for the neoclassical synthesis is irremediably damaged by the capital critique, standard macroeconomic analysis might remain of some use as a policy tool if investment behavior did in fact exhibit a rough empirical regularity of the marginalist type. But the evidence on the relationship between investment spending and the interest rate is not encouraging for adherents to the neoclassical hypothesis (Eisner 1968, p.191). Tinbergen's results (1938–39) suggest that there is no clear and consistent inverse relationship between interest rates and investment. A well-known study by Meade & Andrews (1938), in which businessmen were questioned about the influence of interest rates on their decisions, found that interest rates were not a significant determinant of investment behavior.

Some sectors, such as construction, *are* highly sensitive to financing costs; but this sensitivity simply reflects the elasticity with which the sector is able to take advantage of profit opportunities made possible by short-period price differentials (e.g. between the cost of construction and the price received when the building is sold or leased). The particular organization of the markets in which each such sector operates may account for a fairly consistent empirical relationship between interest rates and investment; but there is no reason to suppose that once the supply of, e.g., housing had adjusted to the point where its price was just sufficient to provide builders with a normal rate of return investment in housing would remain unchanged at the given interest rate.

There is, moreover, room for different views on the connection between investment and the financial environment in which firms operate. Marglin (1984, pp.88–93) argues for example that investment can crowd-out consumption, rather than the other way round, because the corporate sector—the capitalist class—has privileged access to finance; thus there is no real financial constraint on investment. Kalecki (1969, pp.91-5), on the other hand, has pointed out that the current value of a firm's assets sets a practical limit on the amount which it may borrow: "The most important prerequisite for becoming an entrepreneur is the *ownership* of capital" (cf. also, as we have mentioned, Mill 1844b, p.110). Similar constraints, he continues, limit the firm's ability to expand by issuing additional shares of common stock.

Other considerations are likely to exert a comparable influence on investment. The competitive conditions within which firms operate impose certain broad requirements upon firms regarding research and development expenditures and investment—requirements that can be ignored only at the risk of a loss of market share. In the mathematical formulation (14) these influences are captured in the parameters $\boldsymbol{\alpha}$, which, since they carry equal weight with the induced components of investment, can no longer be left unexplained. The corporate cultures and the managerial philosophies of different industries or firms will also have a substantial impact on investment decisions. Expectations, of course, cannot be ignored. But if they are to form part of any real explanation they need to be grounded in observable phenomena, such as the age composition of the population, the pattern of income distribution, the geographical distribution of the population, or the ability of sellers to manipulate preferences through advertising.

A useful explanation of investment, therefore, entails much more than the econometric evaluation of the significance of the usual independent variables. A more flexible interdisciplinary approach, which can take account of historical evidence and can exploit the vast literature on the sociology of organizations, is required. Any explanation of investment along these lines must be specific to particular historical and institutional circumstances. But what might at first appear to be a lamentable loss of generality is in fact no loss at all; for the preceding discussion suggests that the generality of orthodox investment theory is spurious. The investigation of historical and social processes in all their complexity and richness, is bound to produce conclusions that are less clear-cut and more difficult to interpret than the results derived from orthodox theory.