PART IV

THE REEMERGENCE OF THE SOCIAL SURPLUS MODELS, 1930 – 1970

CHAPTER 9

WASSILY LEONTIEF AND INPUT-OUTPUT ANALYSIS

Introduction

In 1973 the Royal Swedish Academy of Sciences awarded Wassily Leontief the Nobel prize for the development of the input-output method and for its application to important economic problems. Leontief first began his work on input-output analysis in the early 1930s; however Leontief was exposed to input-output types of analyses many years prior to this. On the one hand Leontief had been exposed to Quesnay's *Tableau Economique* and Marx's reproduction models before 1935 and probably before he left Russia in the mid-1920s; moreover he was also exposed to the primitive Soviet planning techniques that were being developed in the early 1920s; finally in the early 1930s he was a member of the "Kiel Group" (who we shall discuss shortly) in which some of its members were involved in developing Marx's reproduction models. His first preliminary papers announcing his work appeared in 1936 and 1937; and his definitive monograph, *The Structure of American Economy 1919-1929* appeared in 1941. In this monograph Leontief developed a closed input-output system; however he quickly discovered that the closed system was inappropriate for studying the impact of external events and disturbances on the level of economic activity. Thus in 1944 he published another article in which he introduced the open input-output system and showed how it could be used to estimate the effect of postwar reconversion on the pattern of economic activity. Later he extended his input-output analysis to international trade, the inflationary process, regional economics and environmental economics.

Introduction to an Input-Output System

To establish an input-output system, a descriptive table of the economy called a *transaction table*, needs to be introduced. Its entries are in value terms and include all the various economic flows within the economy for a given period of time. The table can be written in the following manner.

	Commodity 1	Commodity 2	Commodity 3	Total Value of output
Industry 1	$q_{11}p_1$	$q_{12}p_2$	$q_{13}p_3$	Q_1p_1
Industry 2	$q_{21}p_1$	$q_{22}p_2$	$q_{23}p_2$	Q_2p_2
Industry 3	$q_{31}p_1$	$q_{32}p_2$	$q_{33}p_3$	Q_3p_3

a. Let us first look at the first two industries and commodities:

(1) q_{jj} is read as the amount of commodity *j* (which comes from the j^{th} industry) needed to produce Q_i amount of commodity *i* in industry *i*. Thus q_{ij} is a physical amount of commodity *j*. For example, q_{21} reads as the amount of commodity 1 needed to produce Q_2 amount of commodity 2 in industry 2.

(2) p_j is the price of commodity *j*. Therefore $q_{11}p_1$ and $q_{12}p_2$ (and $q_{21}p_1$, $q_{22}p_2$) represents the cost of using commodity 1 and 2 in the production of commodity 1.

b. Now let us consider the third industry and commodity:

(1) The third commodity Leontief defined as capital and labor services and p_3 was their price. Thus $q_{13}p_3$ (and $q_{23}p_3$ and $q_{33}p_3$ consists of the costs of using capital and labor services and is denoted as value added.

(2) Industry 3 consists of the inputs and costs incurred in the production of capital and labor services. This industry is generally called the household industry.

c. Now we are in the position to delineate two essential properties of the transaction table:

(1) The first property:

$$q_{11}\mathrm{p}_1 + q_{12}p_2 + q_{13}p_3 = Q_1p_1$$

$$q_{21}p_1 + q_{22}p_2 + q_{23}p_3 = Q_2p_2$$

$$q_{31}p_1 + q_{32}p_2 + q_{33}p_3 = Q_3p_3$$

This property simply says that total costs (including value added) equals total revenue.

(2) The second property:

$$q_{11}p_1 + q_{21}p_1 + q_{31}p_1 = Q_1p_1$$

$$q_{12}p_2 + q_{22}p_2 + q_{32}p_2 = Q_2P_2$$

$$q_{13}p_1 + q_{23}p_3 + q_{33}p_3 = Q_3p_3$$

This property says that the total cost of using commodity *j* equals the total revenue of industry *j*

(3) A corollary to the second property is that the total output of any industry is completely distributed among the 3 industries as inputs:

$$q_{11} + q_{21} + q_{31} = Q_1$$

$$q_{12} + q_{22} + q_{32} = Q_2$$

$$q_{13} + q_{23} + q_{33} = Q_1$$

d. So far the transaction represents a point in time - a snapshot of the economy. However Leontief wanted to use it analyze economic events over time; therefore the assumed that the production coefficient which are defined as q_{11}/Q_1, q_{12}/Q_1, etc. are stable. That is if Q_1 increases by 10% then q_{11}, q_{12}, and q_{13} also increases by 10% (Empirical support for this is not there).

With the introduction of production coefficients, we can introduce a price model and a quantity model based on the transaction models:

(1) Price model

Let $a_{ij} = \frac{q_{ij}}{Q_i}$ reads the amount of commodity j needed to produce one unit of commodity i.

$$a_{11}p_1 + a_{12}p_2 + a_{13}p_3 = p_1$$

$$a_{21}p_1 + a_{22}p_2 + a_{23}p_3 = p_2$$

$$a_{31}p_1 + a_{32}p_2 + a_{33}p_3 = p_3$$

(2) Quantity model

$$a_{11}Q_1 + a_{12}Q_2 + a_{13}Q_3 = Q_1$$

$$a_{21}Q_1 + a_{22}Q_2 + a_{23}Q_3 = Q_2$$

$$a_{31}Q_1 + a_{32}Q_2 + a_{33}Q_3 = Q_3$$

Closed Input-Output Model

a. A closed model is one in which all the output is absorbed by the producing industries. (This is implied in the properties noted above.)

b. Price model

$$a_{11}p_1 + a_{12}p_2 + a_{13}p_3 = p_1$$

$$a_{21}p_1 + a_{22}p_2 + a_{23}p_3 = p_2$$

$$a_{31}p_1 + a_{32}p_2 + a_{33}p_3 = p_3$$

(1) To solve for prices, we only need to assume that $p_1 = 1$ and solve for p_2 and p_3 in terms of p_1.

(2) Note prices can be obtained in a manner similar to that we used in the sections on Ricardo and Smith and that supply and demand do not have to be invoked to determine prices.

c. Quantity model

$$a_{11}Q_1 + a_{12}Q_2 + a_{13}Q_3 = Q_1$$

$$a_{21}Q_1 + a_{22}Q_2 + a_{23}Q_3 = Q_2$$

$$a_{31}Q_1 + a_{32}Q_2 + a_{33}Q_3 = Q_3$$

(1) To solve for quantities, we assume that $Q_1 = 1$ and solve for Q_2 and Q_3 in terms of Q_1.

(2) This means that our solutions are in terms of relative quantities meaning that they are uniquely tied to the level of Q_1.

d. Examples

(1) Price model

$$\frac{1}{2}p_1 + \frac{1}{3}p_2 + \frac{1}{4}p_3 = p_1$$

$$\frac{1}{3}p_1 + \frac{1}{3}p_2 + \frac{1}{4}p_3 = p_2$$

$$\frac{1}{6}p_1 + \frac{1}{3}p_2 + \frac{1}{2}p_3 = p_3$$

Letting $p_1 = 1$ then $p_2 = 5/6$ and $p_3 = 8/9$

(2) Quantity model

$$\frac{1}{2}Q_1 + \frac{1}{3}Q_2 + \frac{1}{6}Q_3 = Q_1$$

$$\frac{1}{3}Q_1 + \frac{1}{3}Q_2 + \frac{1}{3}Q_3 = Q_2$$

$$\frac{1}{4}Q_1 + \frac{1}{4}Q_2 + \frac{1}{2}Q_3 = Q_3$$

Letting $Q_1 = 1$ then $Q_2 = 1$ and $Q_3 = 1$ (an odd case). Thus if $Q_1 = 10$, then so must Q_2 and Q_3.

Open Input-Output Model

There is a significant difference between the first two industries and the household industry. In the former case, the production coefficient can really be considered as technical datum, while in the latter case, they can be considered as being amenable to economic decision-making. To make this point clear, let us take a closer look at the 'production' side of our above model:

$$\begin{array}{cc|c} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \hline a_{31} & a_{32} & a_{33} \end{array}$$

(1) The set of production coefficients a_{11}, a_{12}, a_{13} are technical datum since they are based on the use of manufactured goods to produce manufactured goods.

(2) a_{13} and a_{23} represent the per unit flow of capital and labor services used to produce commodities 1 and 2. In addition $a_{13}p_3$ and $a_{23}p_3$ represents the *value added* to the production of each good. Since value added consists of wages and profits, then it is quite easy to see that it is amenable to economic decision making such as workers striking to increase wage rates or capitalists increasing their profit margins. This can easily be seen if $a_{23}p_3$ (or $a_{13}p_3$ is divided into wages w_2 and profits π_2 - $a_{23}p_3 = w_2 + \pi_2$

(3) a_{31} and a_{32} represents the per unit flow of commodities one and two used to produce the capital and labor services. *I*o put it a different way a_{31} and a_{32} represent amount of goods 1 and 2 which can be invested in plant and equipment (hence used to 'produce' capital services) and used to feed the workers (hence invested in manufacturing labor services). Therefore $a_{31}Q_1$ and $a_{32}Q_2$ represent the total amount of goods 1 and 2 which can be used for a variety of investment projects and thus are amenable to economic decision making.

<u>Open Price Model</u>

Let us consider the implications of the above discussion in terms of our price model. Because the value added is amenable to economic decision making, it is assumed to be known *exogenously*. Therefore our model takes on the following form:

$$a_{11}p_1 + a_{12}p_2 + w_1 + \pi_l = p_1$$

$$a_{21}p_2 + a_{22}p_2 + w_2 + \pi_2 = p_2$$

With w_1, w_2, π_1, π_2 known, it is easy to solve for p_1 and p_2.

Example:

Let $w_1 = 1 = w_2$ and $\pi_1 = \pi_2 = 2$; also, let $a_{11} = .5$, $a_{12} = .3$, $a_{21} = .2$, $a_{22} = 1$. Thus we have:

$$.5p_1 + .3p_2 + 1w + 2\pi = p_1$$

$$.2p_1 + .1p_2 + lw + 2\pi = p_2$$

Now let us consider what happens when wages and profits are increased exogenously:

(a) wages increase $w_1 = w_2 = 2$ then $p_1 = 12.30$ and $p_2 = 7.17$

(b) profits increase $\pi_1 = \pi_2 = 3$ then $p_1 = 12.30$ and $p_2 = 7.17$

As can be seen in these cases, an increase in either wages or profits increases prices. *Moreover these prices ensure that the wage and profits aimed at are obtained*.

(c) wages increase $w_1 = 2$, $w_2 = 1$, then $p_1 = 11.69$ and $p_2 = 6.15$

(d) profits increase $\pi_1 = 3$, $\pi_2 = 2$, then $p_1 = 11.69$ and $P_2 = 6.15$

In these cases a rise in wages and profits in one industry not only raises prices in these industries, but also raises prices in the second industry. This is because of the interdependent nature of the two industries. This also shows how price increases in one industry can affect all the prices in the economy.

Open Quantity Model

Because investment projects are in a sense imposed upon the economy we can say that they are determined exogenously. Moreover, investment projects require goods which are not being used immediately in production – that is the required *surplus* goods. Therefore when specifying that the amount of surplus goods wanted are Y_1 and Y_2, the quantity model is solving for Q_1 and Q_2 which will produce that amount of the surplus. The quantity model in this case takes on the following form:

$$a_{11}Q_1 + a_{21}Q_2 + Y_1 = Q_1$$

$$a_{12}Q_1 + a_{22}Q_2 + Y_2 = Q_2$$

With Y_1 and Y_2 known, Q_1 and Q_2 can be solved for.

Example:

Let $a_{11} = .5$, $a_{12} = .3$, $a_{21} = .2$, $a_{22} = .1$, $Y_1 = 10$, $Y_2 = 10$. Then we have:

$$.5Q_1 + .2Q_2 + 10 = Q_1$$

$$.3Q_1 + .1Q_2 + 10 = Q_2$$

Solving for Q_1 and Q_2, we find that $Q_1 = 28.2$, $Q_2 = 20.5$

Now let us consider what happens when Y_1 and/or Y_2 is increased

(a) jf Y_1 and Y_2 are both increased by 10%, then $Q_1 = 31.02$ and $Q_2 = 22.56$

(b) if only Y_1 is increased by 10%, then $Q_1 = 30.51$, $Q_2 = 21.28$.

In either case output is increased, in the second case because of the interdependent nature of the economy.

The Keynesian Multiplier in the Quantity Model

Let us look at a special case of our quantity model where $Y_1 = Y_2 = 1$

$$.5Q_1 + .2Q_2 + 1 = Q_1$$

$$.3Q_1 + .1Q_2 + 1 = Q_2$$

Solving we get $Q_1 = 2.82$, $Q_2 = 2.05$. From the answer we see that to produce one unit of each of the investment goods, $Q_1 = 2.82$ and $Q_2 = 2.05$ must be produced. This result is somewhat strange because at first sight it would appear that total output should only be $Q_1 = 1.8$ and $Q_2 = 1.3$ which consists of 1 unit of investment good each plus the direct inputs needed to produce them. However let us look at this again:

(i) to produce $1Y_1$ and $1Y_2$ we need $.8 = Q_1$ and $.3 = Q_2$

(ii) however to produce $.8 = Q_1$ and $.3 = Q_2$ we need $Q^*_1 = .49$ and $Q^*_2 = .19$

(iii) now to produce $Q^*_1 = .49$ and $Q^*_2 = .19$ we need $Q^{**}_1 = .111$ and $Q^{**}_2 = .117$

(c) adding this up we get

$$Q1 = 1 + .8 + .49 + .111 + \ldots$$

$$Q2 = 1 + .3 + .19 + .117 + \ldots$$

Thus if investment is increased by one unit of each good, then total output will increase by more than that one unit (or even if you add in the direct inputs). This is what the Keynesian multiplier is all about.

Two Open Models

(1) Let $w_1 + \pi_1 = v_1$ (for value added) and $w_2 + \pi_2 = v_2$

(2) Thus our model becomes:

$$a_{11}p_1 + a_{12}p_2 + v_1 = p_1$$

$$a_{21}p_1 + a_{22}p2 + v_2 = p_2$$

(3) Now let us multiply the first row by Q_1 and the second row by Q_2, then we get:

$$\begin{aligned} a_{11}Q_1p_1 + a_{12}Q_1p_2 + v_1Q_1 &= p_1Q_1 \\ a_{21}Q_2p_1 + a_{22}Q_2p_2 + v_2Q_2 &= p_2Q_2 \end{aligned}$$

or

$$\begin{aligned} q_{11}p_1 + q_{12}p_2 + v_1Q_1 &= p_1Q_1 \\ q_{21}p_1 + q_{22}p_2 + v_2Q_2 &= p_2Q_2 \end{aligned}$$

Now rearranging it we get

$$\begin{aligned} v_1Q_1 &= p_1Q_1 - q_{11}p_1 - q_{12}p_2 \\ v_2Q_2 &= p_2Q_2 - q_{21}p_1 - q_{22}p_2 \end{aligned}$$

(4) Now $v_1Q_1 + v_2Q_2$ equals gross value added or national income; moreover the right hand side when added together equals net national product. Thus we see that NI = NNP

(5) Since NNP consists of the value amount of our surplus goods and since prices are independent of the level of output, then increasing investment increases NI (or if we increase the dollar amount of investment, NI will increase but so will the physical output of the investment

goods). On the other hand increases in wages and profits also increase NI and as well as NNP. Let us investigate the points a little bit closer.

(a) Let us rewrite v_1 and v_2 as $w_1 + \pi_1$ and $w_2 + \pi_2$; thus we have $(w_1 + \pi_1)Q_1 + (w_2 + \pi_2)Q_2 = Y_1p_1 + Y_2P_2$

(b) If Y_1 and/or Y increase then Q_1 and Q_2 increase with no change in prices; or wages and profits; if w or π increase p_1 and p_2 increase but with no change in output or investment.

(c) Thus if one's object is to increase employment via increasing NI then you must concentrate on Y_1, Y_2 (or investment) as Keynes (and Kalecki) argued. On the other hand, if you want to increase employment via increasing NI by increasing profits share in NI then the only result will be higher prices, a larger share of national income for the capitalist, and no change in the level of employment. This second way is the typical neoclassical argument of savings $\rightarrow$ investment $\rightarrow$ greater employment and we see that it is false in this context. As we shall see when we study Michal Kalecki and Joan Robinson, that trying to raise NI and employment by concentrating on profits will lead to the exact opposite result; whereas on the other hand, if you concentrate on investment, then NI, employment, and profits will increase.

CHAPTER 10

TRANSFORMATION PROBLEM REVISITED

In his *Theory of Capitalist Development* (1946), Paul Sweezy drew the attention of speaking/reading economists to the transformation problem as dealt with by Bortkiewicz. As a result new attention began to be focused on the problem so that between 1946 to 1960 many articles were written on the topic. Out of this attention on the transformation came two important results: (1) attention was again directed towards the notion of the surplus, and (2) the methodology used to carry out the analysis adopted a more of an 'Leontief approach' implying that Marx's models of reproduction and Leontief's input-output models are very similar.

Winternitz

The article which really started off the new discussion of the transformation problem was by J. Winternitz and published in 1948. In reviewing Bortkiewicz's article, Winternitz leveled three criticisms at it. First he complained that Bortkiewicz restricted his analysis by assuming 'simple' reproduction. Rather he argued that solutions could be obtained even if $a_1 \neq C, a_2 \neq V$ and $a_3 \neq S$. Secondly, he claimed that Bortkiewicz's *numeraire* assumption was unMarxian and that it should be replaced by the assumption that total price-values equals total labor values (i.e. $\sum a_i p_i = \sum a_i$). Lastly, he argued that the *numeraire* commodity need not be located in sector three, thus justifying his approach to closing the system.

b. Given his complaints, Winternitz system of equations is as follows:

Sector one: $(c_1 p_1 + v_1 p_2)(1+r) = a_1 p_1$

(1) Sector two: $(c_2 p_1 + v_2 p_2)(1+r) = a_2 p_2$

Sector three: $(c_3 p_1 + v_3 p_2)(1+r) = a_3 p_3$

$$a_1 p_1 + a_2 p_2 + a_3 p_3 = a_1 + a_2 + a_3$$

The implications derived from this are as follows. First although total price-values equals total labor values, individual price-values differ from their labor values. However this result shows that the transformation problem in the formal sense of linking value to prices is a mathematical trivial exercise. Second, since the results are independent of simple reproduction, they are also independent of the modeling approach of dividing the economy into three branches.

Francis Seton

The Bortkiewicz - Winternitz approach to the transformation is special in that it starts with labor values and then goes to prices. Obviously it is also possible to start with prices and go to values. Each of these approaches came into play after Winternitz's article. The ultimate analysis of the transformation problem using the value to prices approach was done by F. Seton in 1957. Seton generalizes Winternitz's system of equations to include n industries (instead of the usual three industry sector models). That is Seton simply posited a Leontief model of the following form:

$$(k_{11} p_1 + k_{12} p_2 + \ldots\ldots + k_{1n} p_n)(1+r) = TV_1 p_1 [rTV_1 = s_1]$$

(2) $$(k_{21} p_1 + k_{22} p_2 + \ldots\ldots + k_{2n} p_n)(1+r) = TV_2 p_2 [rTV_2 = s_2]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$(k_{n1} p_1 + k_{n2} p_2 + \ldots\ldots + k_{nn} p_n)(1+r) = TV_n p_n [rTV_n = s_n]$$

where k_{ij} equals $c_{ij} + v_{ij}$ and represents the amount of commodity; reckoned in

embodied labor used in the production of commodity i;

r is the rate of profit;

TV_i is the total labor value of the output of commodity i; and

p_i is the price of commodity.

Prices can now be solved for, but the specific solution for them will depend on the assumption invoked. Seton introduced three different assumptions:

(1) Assume a commodity numeraire – such as Bortkiewicz did;

(2) Assume that total labor value equals total price – that is

$$TV_1 + ... + TV_n = TV_1 p_1 + ... + TV_n p_n$$

(3) Assume that total profit equals total surplus value – that is

$$s_1 + ... + s_n = rTV_1 p_1 + ... + rTV_n p_n$$

Seton noted that the choice of assumption for solving for prices had no objective basis mathematically. However, because Marx assumed that the second and third assumption to hold simultaneously and because of the non-mathematical objective basis for choosing any assumption, Seton decided to discover the conditions under which all three of the postulates would hold simultaneously. Working with the traditional three industry model

$$(k_{11} p_1 + k_{12} p_2)(1 + r) = TV_1 p_1$$

$$(k_{21} p_1 + k_{22} p_2)(1 + r) = TV_2 p_2 \tag{3}$$

$$(k_{31} p_1 + k_{32} p_2)(1 + r) = TV_3 p_3$$

where $\begin{aligned} k_{i1} &= c_{i1}; and \\ k_{i2} &= v_{i2} \end{aligned}$

Seton found that if $k_{31} : k_{32} : TV_3 = k_{11} + k_{21} : k_{12} + k_{22} : TV_1 + TV_2$ and if simple reproduction holds (that is $rTV_1 p_1 + rTV_2 p_2 + rTV_3 p_3 = TV_3 p_3$) then the three assumptions will hold simultaneously.

These results should not appear surprising for the above assumptions are similar to the ones made in our discussion of Marx and made by Bortkiewicz. What is important to note is that prices are not necessarily equal to their embodied labor values. Thus Seton investigation shows that the uniformity of the organic composition of capital can be weaken somewhat and still achieve important results – that is results which were not inconsistent with the results Marx hypothesis. For a more mathematical presentation of Seton, see the Appendix.

Prices to Values

The second approach to the transformation problem – that is from prices to value – began to emerge in the early 1950s as a result of extensive analysis of Leontief's input-output models. The argument along this line was first proposed by B. Cameron (1952) and N. Georges – Roegen (1950). At this primitive stage the modeling to the form of

$$\begin{aligned} 280p_c + 12p_m + 60L_w &= 575p_c \\ 120p_c + 8p_m + 10L_w &= 20p_m \end{aligned} \tag{4}$$

We already know that if the rate of profit is zero, prices will be proportional (or equal) to embodied labor values. However it quickly was shown that the special assumption of transforming prices into values (r = 0) was not really needed. But for our purposes the actual procedure used is of no importance. What is of importance is that economists had to use 'surplus' models to deal with the problem.

Appendix

Seton first generalizes Winternitz's system of equations to include n industries which, in turn, represented an indecomposable technology matrix. That is, instead of working with the three department models, Seton simply posited a 'Leontief' matrix of the following form:

(5) $$\begin{bmatrix} k_{11}k_{12}\ldots\ldots k_{1n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ k_{n1}k_{n2}\ldots\ldots k_{nn} \end{bmatrix} \begin{bmatrix} p_1 \\ \ldots \\ p_n \end{bmatrix} = \mu \begin{bmatrix} N_1 0 \\ 0 \ddots N_n \end{bmatrix} \begin{bmatrix} p_1 \\ \ldots \\ p_n \end{bmatrix} \text{ or}$$

(6) $K_p = \mu N_d p$

where k_{ij} represents the amount of commodity j reckoned in embodied labor used in the production of commodity i_j

μ is $1-\pi$ where π is the ratio of profit to total value of output (obviously π represents the surplus value); and

N_i is the total labor value of the output of commodity i.

Seton rearranged (6) by pre-multiplying by N_d and then putting into the form of a homogenous set of equations:

(7) $kp = \mu p$ or $(k - \mu I)p = 0$

where $k = N_d^{-1} k$

Assuming k is semi-positive and indecomposable and $\lambda_m(k) < 1$, solution for μ is obtainable while the solution for p will depend on the assumption invoked.

At this juncture, Seton introduced the "postulates of invariance", by which p could be solved for. One postulate is to assume a commodity *numeraire* – such as Bortkiewicz assumed. A second postulate is to assume that total labor value equals total price (i.e. $N'_p = N'_1$) or all prices add up to one (i.e. $N'N^{-1}{}_{dp} = N'N^{-1}{}_{d1}$ or $1'_p = 1'1 = 1$). In this case prices will generally deviate from their values (except under special conditions). The third postulate is to assume total profit (in price terms) equals total surplus value (i.e. $s'1 = \pi N'_p$). Seton noted that the choice of postulate for solving for prices had no objective basis mathematically. However, because Marx assumed the second and third postulate to hold simultaneously, and because of the non-mathematical objective basis for choosing any postulate, Seton decided to discover the conditions under which all three of the postulates hold simultaneously.

Working with the three department model where $k_{i1} = c_i / a_i$ and $k_{i2} = v_i / a_i$, we have

(8) $$\begin{bmatrix} k_{11} k_{12} 0 \\ k_{21} k_{22} 0 \\ k_{31} k_{32} 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \mu \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

Assuming "proper conditions", we find the rate of profit determined solely in the basic commodities sector and a positive vector of prices which are determined by the basic sectors. Now if we assume that $k_{31} : k_{32} : N_3 = \sum k_{i1} : \sum k_{i2} : \sum N_i$, and simple reproduction (that is $\sum s_i p_i = a_3 p_3$), then all three postulates hold simultaneously. This can be seen in the following manner:

(i) The first assumption implies that N'**1**/s'**p** = N'**p/s'1**p3

(ii) The second assumption implies that $s'p = s'_1\ p_3$

(iii) Substituting the results of (ii) into (i) we have $N'_1 / s'_1 = N'_p / s'_p$ which implies that the three postulates are simultaneously fulfilled if $p_3 = 1$

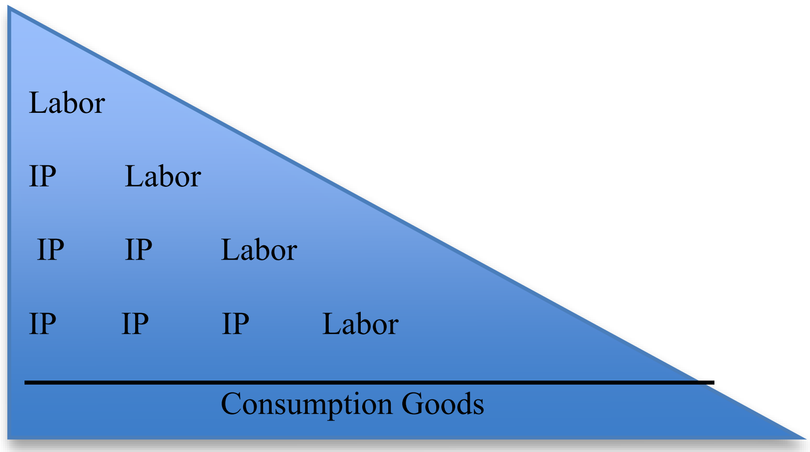
Lastly Seton showed that $p_i >< 1$ if $K_{i1} / k_{i2} >< \sum k_{i1} / \sum k_{i2}$, if the second postulate is assumed.

CHAPTER 11

KIEL GROUP AND THE BURCHARDT PRODUCTION-PRICE MODEL

With the impact of the Great Depression of the 1930s went an increased interest in the theory of capital and interest, in the hope of producing superior policies to help ameliorate economic fluctuations. This re-examination centered on two aspects of 'Austrian' capital theory: the possibility of extending the concept of 'period of production' into a theory of the trade cycle; and, whether the assumption of a stationary state upon which such capital theory rested, could be removed without serious damage to the theory. The 'Austrian model' of the structure of production originally was formulated by Bohn-Bawerk and in the 1930s it was pushed by F.A. von Hayek in his book *Prices and Production*. In the book production is depicted as a triangular diagram. At the apex of the triangle the original factor (labor) alone produce the first intermediate products. These products then move downwards through the triangle and in doing so gradually increase in value as successive applications of labor is applied. The height of the triangle can therefore depict the absolute period of production, while the base indicates the quantity of labor employed. The areas of the triangle represented the total stock of 'intermediate products (capital) in existence at any moment of time.

Example



While many economists engaged in controversy with the Austrian model, there is a small group of economists associated with Kiel University who joined in the controversy by reconstruction the reproduction models of Marx. Of special interest will be the work of Fritz Burkhardt and Adolph Lowe.

Kiel Group

German economics of the early 20th century was still dominated by historicism and social reformers. The so-called 'marginal' revolution of the 1870s was still to have its impact on German economists. Even by the outbreak of WWI it was difficult to find serious exponents of 'theory', of either a non-classical or classical type. The tumultuous post-war years helped change this picture, with their demand for answers to the inflation of the early 1920s, with the need to provide theoretical arguments against crippling reparation payments, and with a growing debate about the pros and cons of nationalization. Theory was to remain no longer the almost exclusive preserve of the German Marxists. With the setting up of the Kiel Institute, theoretical research was given a special boost. In such an historical environment it is little wonder that German economists were particularly interested in economic dynamics, the equilibrium of neo-classical economics was not reflected in the economy they were trying to stabilize.

The major research interest of the group was on the construction of a theoretical model of cyclical growth, based on the problems of world industrialization, but capable of serving as a basis for policy predictions. Burchardt and others found little assistance in the theory of value and distribution the prevailing in neoclassical theory. Their attention was naturally directed back to classical and Marxian analysis, and to a critique of Schumpeter's schema work on economic development. Burchardt began with Schumpeter's scheme, rejected Schumpeter's assertion that 'dynamics', as a theory of development or growth, was less 'exact' and unrelated to the theory of stationary equilibrium. He also began a serious study of the then prevailing explanations of the trade cycle. Commencing with a thorough survey of such explanations, from David Hume through to Wicksell, he presented a refutation of exclusively monetary explanations of the trade cycle. 'Real' factors, he argued, were more central to the problems, especially the impact of technical change. This orientation turned his attention back to work that had combined both value and physical aspects of the production process, of a nature not found in the work of those at Cambridge or Lausanne. Quesnay, Marx and Bohm-Bawerk, he saw at the best starting points.

Burchardt Production-Price Model

Buckhardt development of a theoretical model of growth, which is based on production, was done in 3 steps: the first step involved a long negative critique of Bohm-Bawerk's model of production; the second step involved an analysis of Marx's models of reproduction; and the third step involved a contrasting of the linear approach of the Austrians with the emphasis Marx placed on the circularity of the production process. Burchardt disputed the picture provided by Bohm-Bawerk of 'intermediate products' steadily moving down the strictly one-way road of the process towards their final goal, consumption. He argued that the reproduction and expansion of the stock of fixed capital goods in a state of full resource utilization cannot be explained by Bohm-Bawerk's approach. Simply tracking the technical process of production back to some original combination of natural resources and labor does not explain the reproduction of fixed capital. Such capital, though itself an input, he claimed can only be maintained and expanded with the assistance of a circular process in which fixed capital goods also act as inputs. Burchardt does not question the ability of Austrian analysis to deal with the problem of working capital on the highest stage of production. If on the highest stage a stock of fixed capital goods is added to the original inputs of labor and natural resources the downward flow to the final stage of finished output properly describes the structure of working capital. Nevertheless, the Austrian model must be supplemented with the Marxian schema of expanded reproduction, which clearly illustrates the reproduction of fixed capital goods.

The Two-Sector Burchardt Production-Price Model

Schema of Production

The Burchardt production model consists of two industries or sectors - the machine good sector and the consumption good sector. The schema of production within each sector takes place along classical lines. Thus if we assume that there is only one stage of production within each sector and that the original inputs consists of labor and raw materials not produced within the economy, the direct input/working capital production schema of the model takes the following form:

(1) $L_m + RM_m \dashrightarrow Q_m$

$L_c + RM_c \dashrightarrow Q_c$

Where L_m is the amount of labor needed to produce Q_m number of machines;

RM_m is the amount of raw materials needed to produce Q_m number of machines;

L_c is the amount of labor needed to produce Q_c number of consumption goods; and

RM_c is the amount of raw materials needed to produce Q_c number of consumption goods.

As depicted in the production schema, labor directly uses the raw materials to produce the output. In its productive effort, labor is also assisted by machines; but the machines themselves are not part of the direct inputs (or working capital) used in production. Hence behind each industry is a bank of machines used for production.[1] Therefore the production schema can be augmented in the following manner:

(2) M_m: $L_m + RM_m \dashrightarrow Q_m$

M_c: $L_c + RM_c \dashrightarrow Q_c$

where M_m is the number of machines used in the production of Q_m number of machines; and

M_c is the number of machines used in the production of Q_c number of consumption goods.

Production within the augmented schema takes the form of a recipe in that each machine is associated with a fixed amount of labor and a fixed amount of raw materials, the combination

[1]The machine sector produces a single kind of machine with machine-making and consumption good-making attributes. But once they get assigned to a specific sector the loose the other attribute and therefore become specific to take sector.

which produces a fixed amount of output. Therefore the production schema can be recast as follows:

(3) $M_m(a_m + b_m) \dashrightarrow M_m q_m = Q_m$

$M_c(a_c + b_c) \dashrightarrow M_c q_c = Q_c$.

where a_m is the amount of labor associated with a machine in the machine sector;

a_c is the amount of labor associated with a machine in the consumption good sector;

b_m is the amount of raw materials associated with a machine in the machine sector;

b_c is the amount of raw materials associated with a machine in the consumption good sector;

q_m is the number of machines produced by a machine plus its complement of labor and raw materials and $q_m > 1$; and

q_c is the number of consumption goods produced by a machine plus its complement of labor and raw materials.

If we divide each equation by its output, we will get the *labor and raw material production coefficients* for each sector:

(4) $l_m + rm_m \dashrightarrow 1$ machine

$l_c + rm_c \dashrightarrow 1$ consumption good

where l_m is the amount of labor needed to produce one machine;

l_c is the amount of labor needed to produce one consumption good;

rm_m is the amount of raw materials needed to produce one machine; and

rm_c is the amount of raw materials needed to produce one consumption good.

From equation (3) we can also deduce the machine-output ratio for each sector as $1/q_m$ and $1/q_c$.

Production-Price Model of the Economy

Working with our basic schema of production (equation 4), it can be transformed into a model of the economy with the introduction of wage rates, prices of raw materials, profit mark ups, and quantities of output.

(5) $Q_m(rm_m p_{rm} + l_m w_m)(1 + r_m) = Q_m p_m$

$Q_c(rm_c p_{rc} + l_c w_c)(1 + r_c) = Q_c p_c$

where p_{rm} is the price of the raw material used in the production of machines; and

p_{rc} is the price of the raw material used in the production of consumption goods.

The machine good industry produces machines for its own use and for use in the consumption good industry. The first claim on the machines produced (Q_m) are for replacement of machines worn out in production. We shall assume that the machines used in production wear out after a single production period; therefore out of the Q_m machines produced, M_m and M_c are needed to replace the machines worn out in the machine and consumption good industries respectively. If $M_m + M_c \leq Q_m$, then there is a surplus of machines which can be used to increase the productive capacity of each industry; otherwise the output of machines is just sufficient to replace what is used up in production. Turning to the output of consumption goods, its distribution between capitalists, landlords, and workers depends on the saving propensities of the three classes. Presently we shall assume that landlords and workers spend their entire income on consumption goods so that their saving propensity is zero ($s_l = 0$ and $s_w = 0$), and that capitalists do not spend any of their profits or income on consumption goods so that their saving propensity is unity ($s_c = 0$), which implies that they spend their profits only on machines, that is investment goods. Therefore, under these conditions Q_c is distributed entirely to landlords and workers.

The gross national product (GNP) of the economy is $Q_m p_m + Q_c p_c$. Since by definition, GNP equals gross national income (GNI), it is possible to establish the following relationships:

GNP	GNI
$Q_c p_c$	$Q_m l_m w_m + Q_c l_c w_c$ (Total Wage Income) + $Q_m rm_m p_{rm} + Q_c rm_c p_{rc}$ (Total Landlord Income)
$Q_m p_m$	$Q_m(rm_m p_{rm} + l_m w_m)r_m + Q_c(rm_c p_{rc} + l_c w_c)r_c$ (Total Profits)

That is, the total value of the machines produced equals total profits of the capitalists and that the total value of consumption goods equals the total wage and landlord income. From these relationships we can state the share of income in GNP:

$$(6) \qquad i_s = \frac{\text{Income}}{\text{GNP}} = \frac{Q_m l_m w_m + Q_c l_c w_c + Q_m rm_m p_{rm} + Q_c rm_c p_{rc}}{Q_m p_m + Q_c p_c}$$

If the profit mark ups are the same for both industries, then

$$(7) \qquad i_s = 1/1 + r.$$

Thus the income share would vary inversely with the profit mark up. Hence i_s is determined independently of prices and wage rates; or to put it another way, i_s is determined solely by the capitalists when they determine their profit mark up. When the profit mark ups are different in each industry, the actual income share will be affected by prices and wage rates, but the profit mark up will still ultimately determine i_s. That is, if the profit mark ups increase, the income share will fall without exception; on the other hand, an increase in wage rates will not increase the income share since the profit mark ups protect capitalists profits from the encroachment of wages.

The usual assumption made within these models is that capitalists decide to investment and that these decisions run the economy and determine GNP and the level of employment. This can be shown in the following manner. Since profits equals investments and hence the value of the machines produced, we can state the following:

(8) $$\text{GNP} = \text{Investment} + \text{Income} = \frac{\text{Investment}}{(1 - i_s)}.$$

From this we can carry out two comparative static arguments. The first is that if investment increase, given i_s, then GNP increases; and the second is that if the income share increase, given investment, then GNP increases. To really understand these results, it is necessary to look at their microfoundations; and this will require that we disaggregate the model of the economy into a price model and a quantity model.

Price Model

The price model can be derived from equation (8) by dividing both sides by the quantity of output for each industry:

(9) $$(rm_m p_{rm} + l_m w_m)(1 + r_m) = p_m$$

$$(rm_c p_{rc} + l_c w_c)(1 + r_c) = p_c$$

The prices of the raw materials are determined outside the model, as are the wage rates. The profit mark ups on the other hand are determined within the model, as will be shown below. However, for the moment, for given values for wage rates, prices of raw materials, and profit mark ups, it is possible to solve for p_m and p_c. For any increases in raw material prices, wage rates, or profit mark ups, prices will increase. Moreover, if wage rates increase by 10% for example while the profit mark ups remain constant, then prices will adjust so that the share of costs in the price does not change; this is the micro-explanation for the argument above which stated that changes in wage rates cannot encroach on the share of profits in GNI. In addition, it should be noted that changes in raw material price or wage rate in one industry will not affect the price of output in the other industry; and this conclusion also holds for the profit mark up. Finally, if we assume that the material and labor production coefficients do not change with output, then p_m and p_c do not alter with output.

Quantity Model

To simplify matters, we shall first start with the situation in which the total number of machines produced, Q_m, is equal to the number of machines used in the machine industry and the number of machines used in the consumption good industry; it shall also be assumed that the total output of consumption goods, Q_c, is bought by the landlords and workers. These assumptions means that the economy is unable to grow and is simply reproducing itself. To construct the quantity model with these assumptions, we need to draw upon the machine-output ratios for each sector, $1/q_m$ and $1/q_c$:

(10) $(1/q_m)Q_m + M_c = Q_m$

$$q_cM_c = Q_c.$$

or

(11) $[q_m/(q_m - 1)]M_c = Q_m$

$$q_cM_c = Q_c.$$

Given the above assumptions, values for q_m and q_c are known and given; therefore for any given value for M_c, values for Q_m and Q_c can be determined. If M_c is increased, given q_m and q_c, then both the output of machines and consumption goods will be increased. Moreover, for a given M_c, any reduction in q_m or q_c will lower the total output of machines and consumption goods. Finally, from equation (5), we can see that employment and the use of raw materials are directly related to output.

Let us now return to the relationship between GNP, investment and the income share. Given the above assumptions, physical investment is equal to the total number of machines produced, Q_m, which in turn is determined by M_c; and total investment in money terms is $Q_m p_m$. Hence when it is assumed that total investment has increased in money terms, what is actually occurring is that physical investment has increased, which in terms of the model means that M_c has increased. With increasing M_c, it is clear from equation (11), both Q_m and Q_c have increased; and since prices have not changed, GNP has as a consequence increased. Turning to the income share, we find that, after substituting $[q_m/(q_m - 1)]M_c$ and $q_c M_c$ for Q_m and Q_c respectively and then simplifying, i_s is determined by given technology, wage rates, and raw material prices, and output prices:

$$(12) \qquad i_s = \frac{[q_m/(q_m - 1)]l_m w_m + q_c l_c w_c + [q_m/(q_m - 1)]rm_m p_{rm} + q_c rm_c p_{rc}}{[q_m/(q_m - 1)]p_m + q_c p_c}.$$

Changes in wage rates or raw material prices will change both income and GNP, but because the increase wage costs will be feed through to the price, i_s will not increase significantly. On the other hand, if the profit mark ups are increased, output prices will increase and with wage rates and raw material prices remaining constant, the income share will fall. Because changes in the income share can only be effected by changes in wage rates, prices of raw materials, and profit mark ups, which in turn affect the money value of investment, the analysis of its impact on GNP and employment levels is more complex. For example, if profit mark ups are increased, output prices will increase; consequently, for any given M_c, the money value of investment will increase or conversely for any given money value of investment, M_c, will decline. The above discussion after equation (8) presupposed the latter case, but this case is in fact inappropriately specified. To fully appreciate the complexity of the relationship between GNP and the income share, it is necessary to return to the model of the economy as a whole.

Model of the Economy Again

Given the above assumption, the model of the economy can be delineated as follows:

(a) $Q_m(rm_m p_{rm} + l_m w_m)(1 + r_m) = Q_m p_m$

(b) $Q_c(rm_c p_{rc} + l_c w_c)(1 + r_c) = Q_c p_c$

(13) (c) $[q_m/(q_m - 1)]M_c = Q_m$

(d) $q_c M_c = Q_c$

(e) $Q_m(rm_m p_{rm} + l_m w_m)r_m = (Q_m - M_c)p_m = M_m p_m$

(f) $Q_c(rm_c p_{rc} + l_c w_c)r_c = M_c p_m$.

Equations (13a, b) represent the model of the economy as well as the price model, while equations (13c, d) represent the quantity model. Equation (44e) states that all the profits in the machine industry are spent on purchasing machines to replace those that have worn out; while equation (13f) states that all the profits of the consumption good industry is spent on purchasing machines to replace those that have also worn out. Thus all profits are saved and spent on purchasing investment goods (i.e. machines). The technical givens of the model are rm_m, rm_c, l_m, l_c, q_m, and q_c; while values are assumed for p_{rm}, p_{rc}, w_m, w_c, and M_c. The unknowns of the model include p_m, p_c, Q_m, Q_c, r_m, and r_c. What is significant about this fully specified model of the economy based on the above assumptions is what determines the profit mark ups. In the case of r_m it is technically determined by q_m:

(14) $r_m = 1/(q_m - 1)$.

As for r_c, it is determined by the technical givens of the model as well as the assume values for the wage rates and prices of raw materials:

(15) $$r_c = \frac{rm_m p_{rm} + l_m w_m}{rm_c p_{rc} + l_c w_c} \text{ x } \frac{q_m}{q_c(q_m - 1)}.$$

One consequence of equations (14 and 15) is that the profit mark ups are not affected by the level of physical investment; on the other hand, r_c is affected by the money wage rate in that increasing w_m will increase r_c and hence p_c while increasing w_c will reduce r_c and hence p_c. But what cannot occur is an autonomous change in the profit mark ups. Therefore, in light of equation (8), an increase in investment means a increase in physical investment only and hence an increase in GNP due to an increase in the amount of machines and consumption goods produced. Turning to the income share, we find that since the profit mark ups cannot be arbitrary changes, it cannot be affected by them. To change i_s, it is necessary to change the wage rates (or the prices of the raw materials); but because the changes will affect both the amount of income as well as prices, the overall effect will be minimal. Moreover an increase in the income share by increasing the wage rate will also increase the money value of investment; thus the overall impact of a change in the income share will have little effect on GNP, and what impact it will have will be only in value terms since the physical quantities of output are not affected by such changes. The problem with the arguments following equation (8) is that they we framed in a partial manner where it was implied that changes in i_s was independent of any impact on prices;

and this was shown above not to be possible. After all of this, one can still ask the question of why the income share is a relevant variable for economic analysis. Because it is a synthetic variable, it is not directly needed to solve the above model; in fact it does not add a great deal to understanding the workings of the model.

3. The impact of Burchardt's work was significant with respect to the capital controversy at hand.

(i) first of all it undermined the concepts of absolute period of production, and average period of production;

(ii) second, it was shown that when production was a circular process, the quantity could not be determined independent of the rate of interest (profit)

(iii) the capital intensity of production could not be approximated by the A.P.P.

However, beyond the capital controversy, Burchardt's rehabilitation of the truly classical surplus model (as opposed to the Austrian surplus model) fell on faint ears. In fact, it was only when Sraffa's book <u>Production of Commodities</u> was published did the approach that Burchardt champion become widely known and even somewhat accepted.

Adolph Lowe

Alfred Kahler

Gaitskell, H. T. N.

Fleming, J. M.

Other

CHAPTER 12

PRODUCTION-PRICE AND ACCUMULATION MODELS, 1950S

Michal Kalecki

A. Let us first start a simple input - output model of the economy.

Industry M : $(15p_m + 20p_c)(1 + r_m) + 50w = 50p_m$

Industry C : $(30p_m + 30p_c)(1 + r_c) + 60w = 60p_c$

where p_m is the price of good m (machines);

p_c is the price of good c (corn);

r_m is the profit markup for good m;

r_c is the profit markup for good c; and

w is the wage rate.

From the model it can be seen that when the total output is 50 machines and 60 corn, the total labor force is 110 and that it takes 45 machines and 50 corn to produce the output.

1. From the model we can derive Gross National Product as $50p_m + 60p_c$. Consequently, Net National Product can be obtained by subtracting out the material inputs: NNP = 5 machines and 10 corn. In this case the NNP is the physical surplus of the economy.

2. From the model we can also derive Net National Income which is $110w + r_m(15p_m + 20p_c) + r_c(30p_m + 30p_c)$.

3. In equilibrium, NNI = NNP and this produces the following relationships:

<u>NNI</u>	<u>NNP</u>
Total Wage Bill = 110w	corn goods = $10p_c$

Total Profits = P machine goods = $5p_m$

a. Let us assume a 2-class society of workers and capitalists where workers spend all their income on consumption, capitalists spend part of their income (profits) on consumption and part on investment. Therefore it is possible to write the relationship between profits and capitalists expenditure pattern as:

$P = I + C_c$ where C_c is capitalist consumption. If we further assume that capitalists spend a given percentage of their profits on consumption then we have:

$P = I + C_c$

$C_c = qP$, where q is the given percentage

$P = I + qP = I/(1-q)$

b. Thus we can now rewrite NNI and NNP as follows

NNI	NNP
Wage Bill (W_B)	Workers Consumption (W_c)
Profits (I)	Capitalists Consumption (C_c) Investment (I)

4. Working with identities we can derive the following

a. NNI = P + W or
NNI - W = P or
NNI - [W/NNI] x NNI = P

b. now letting [W/NNI] = a which represents the wage share in NNI we now have:

NNI - aNNI = P or
NNI = P/(1-a)

c. at this stage we can substitute I/(1-q) for P and we get: NNI = I/(1-a)(1-q)

5. At this stage, it is possible to derive some comparative statics results. For a give money value of I if

   (1) a increases then NNI increases - (q remains constant)

   (2) a decreases then NNI decreases - (q remains constant)

   (3) q increases then NNI increases - (a remains constant)

   (4) q decreases then NNI decreases - (a remains constant)

   (5) q increases then P increases

   (6) q decreases then P decreases

B. The above results are derived from macro-identities; thus there is little behavioural content in the results. To rectify this let us develop the micro foundations of the marco results.

1. First let us break down the model of the economy into a price model and quantity model.

   a. Price Model

   $(.3p_m + .4p_c)\ (1 + r_m) + 1w = p_m$

   $(.5p_m + .5p_c)\ (1 + r_c) + 1w = p_c$

(1) since this is an equilibrium model, it is possible to obtain solutions for p_m and p_c by assuming values for r_m, r_c, and w.

(2) assuming $r_m = 10\%$, $r_c = 10\%$, and w = \$1.00, then

   $p_m = \$14.92$

   $p_c = \$20.46$

   b. Output Model

   $.3Q_m + .5Q_c + Q_m^* = Q_m$ = total amount of machines produced

   $.4Q_m + .5Q_c + Q_c^* = Q_c$ = total amount of corn produced

(1) since this is an equilibrium model, it is possible to obtain solutions for Q_m and Q_c by assuming values for Q_m* and Q_c* which represent the physical surplus or the physical NNP.

(2) assuming $Q_m^* = 5$ and $Q_c^* = 10$, then $Q_m = 50$ and $Q_c = 60$.

2. Working with the two models we can derive the following macro economic results:

a. $GNP = 50p_m + 60p_c = (50)(\$14.92) + (60)(\$20.46) = \$1973.60$

b. $NNP = 10p_c + 5p_m = (10)(\$20.46) + (5)(\$14.92) = \$279.20$

c. $NNI = 110w + r_m(15p_m + 20p_c) + r_c(30p_m + 30p_c) = \$279.20$

3. Since workers spend all their income on consumption then workers' consumption $W_c = \$110 = 5.38p_c$. Further, we shall assume that 50% of the remaining corn is consumed by the capitalists, or $C_c = 2.31p_c = \$47.30$. Finally, the remaining corn and all the machines are used for investment purposes: $I = 2.31p_c + 5p_m = \$121.90$.

a. In this case profits can be written as

$$P = I + C_c$$

$$= [2.31pc + 5pm] + [2.31pc]$$

$$= \$121.90 + \$47.30$$

$$= \frac{\$121.90}{1-q} \qquad \text{where } q = .28 = \frac{\$\,47.30}{\$169.20}$$

b.

NNI	NNP
Wage Bill = 110w = \$110.00	W_c = \$110 = 5.38pc
Profits = \$169.20	C_c = \$47.30 = 2.31pc I = \$121.90 = 2.31pc + 5pm

c. Now it is possible to determine the wage share a:

$$a = W/NNI = \frac{\$110.00}{\$279.20}$$

$$= \frac{110w}{110w + (r_m)(Q_m)[.5p_m + .5p_c] + (r_c)(Q_c)[.3p_m + 4p_c]}$$

$$= 30\%$$

Michal Kalecki
Joan Robinson
Nicholas Kaldor
John von Neumann

PART V

THE REESTABLISHMENT OF THE SOCIAL SURPLUS APPROACH, 1960 - 2010

CHAPTER 13

SRAFFA AND THE PRODUCTION OF COMMODITIES BY MEANS OF COMMODITIES