ADVANCED MICROECONOMIC ANALYSIS Professor F. S. Lee
(ECON 5502)
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PROBLEM SET 1

Defining Economics, Methodology, and Models

1. How is Lionel Robbin's definition of neoclassical economics reflected in the methodology of modern neoclassical economics?

2. Compare and contrast Marshall's definition of economics and Robbin's definition.

3. How does Marshall justify that wants/desire can be indirectly measured by price.

4. What is Marshall's of economic laws? Is his view different of the modern view of economic laws?

5. What did Marshall mean by "normal"?

6. What is the importance of *ceteris paribus* assumption for theorizing?

7. What role does deduction play in Marshall and in modern neoclassical theorizing?

8. What role does methodological individualism play in neoclassical theorizing?

9. Can the methodology used by neoclassical economists actually create theories that can explain and/or predict the real world?

10. What is positivism?

11. What is instrumentalism?

12. What roles do models and mathematics play in neoclassical theorizing?

13. Is testing hypotheses the only way neoclassical economists explain the real world?

14. Do neoclassical theories need to be realistic in order to explain/predict economic reality?

15. The methodology used in neoclassical economics ensures that its theories are empirically grounded. Discuss.

PROBLEM SET 2

Theory of Consumer Behavior and Demand

Marshall's Analysis of Demand

1. When and where did Marshall discover the price elasticity of demand?

2. How did Marshall eliminate the income effect from his analysis?

3. Given the following additive utility function $U = (y_1)^{.5} + (y_2)^{.5}$ answer the following questions:
   a. derive the marginal utility of y_1 and y_2.
   b. does this utility function conform to the law of diminishing marginal utility? Explain.
   c. derive the Marshall demand curves for both goods.

4. Discuss Marshall's view of substitutes and complements

5. Marshall remarked that the higher study of consumption must come after, and not before, the main body of economic analysis. Discuss whether the higher study of consumption can actually take place within neoclassical demand theory.

6. A consumer faces a utility function $U = \mu(y_1,y_2)$ and a budget constraint of $p_1y_1+p_2y_2=M$.
   a. State the assumptions that must be made if the utility function is to be a "Marshall" utility function. Show the conditions for consumer equilibrium and prove that it is a utility maximization equilibrium.
   b. State the assumptions that must be made if the utility function is to be a "Hickian" utility function. Show the conditions for consumer equilibrium and prove that it is a utility maximization equilibrium.
   c. Explain why Marshall has a determinant negative sign for $\partial y_1/\partial p_1$ while Hicks has an indeterminant sign for $\partial y_1/\partial p_1$.

7. Discuss the importance of the relationship between wants and economic activities for Marshall's analysis of demand.

Modern Utility and Preference Theory

1. What axioms are needed to establish the existence of a continuous utility function?

2. What is lexicographic preference ordering and what does it imply about the existence of the utility function?

3. What axioms ensure the existence of a utility function whose properties would permit utility maximization subject to a budget constraint?

4. What are the properties of a strictly quasi-concave utility function? What do they imply about the Law of Diminishing Marginal Utility?

5. What does the negative slope of an indifference curve imply about a consumer's tastes for two goods? Would the slope be negative if the individual disliked consuming one of the goods?

6. Let us assume that consumer A orders her preferences lexicographically in the following manner--books, food, clothing, and movies. From the following possible commodity bundles, chose the one which would maximize consumer A's utility: (4, 3, 7, 2) (6, 2, 1, 10) (2, 8, 4, 3) (8, 6, 8,5) (5, 10, 3, 6) (8, 6, 4, 7) (7, 6, 10, 7) (7, 10, 2, 4).

7. What major propositions in standard consumer theory would remain unchanged if smooth indifference curves existed but were not necessarily of the usually assumed convex shape? And which important results would not hold?

8. There are many goods that a consumer does not buy at all. Can we expect the marginal conditions for utility maximization to hold for these goods? Are their marginal utilities likely to be smaller or larger than those of other goods?

9. Show that the income expansion line is a straight line if the utility function is given by $U = (y_1)^{.5}(y_2)^{.5}$.

10. The microeconomic theory of consumer behavior hinges on the concept of subjective individual preference. Individual preferences are either (a) unobservable or (b) observable. Furthermore, individual preferences are either (A) exogenous or (B) endogenous.
    a. Consider the a-A pair of assumptions. What are some theoretical implications of postulating unobserved, exogenous tastes as microeconomists go about constructing their theory of individual consumer behavior?
    b. Select one other pair of assumptions: either a-B, b-A, or b-B. For that pair of assumptions, what are some theoretical implications as microeconomists go about constructing their theory of individual consumer behavior?
    c. In what ways, if any, does the pair of assumptions actually employed affect how one theories about individual consumer behavior?

11. Consider a consumer with a utility function of $U = y_1y_2$. Find the consumer equilibrium position when M = \$15, p_1 = \$5, and p_2 = \$1.

12. Local-nonsatiation and convexity of preferences both rule out “thick” indifference curves. True or False. Discuss.

13. What does lexicographic preference ordering imply about the existence of the utility function?

14. How is utility maximization ensured in modern consumer theory?

Demand Theory

1. Suppose a consumer's utility function is represented by $U = b_1\ln(y_1 - r_1) + b_2\ln(y_2 - r_2)$ where $0 < b_1, b_2 < 1$, $0 < r_1, r_2$, and $b_1 + b_2 = 1$.
   a. Over what values of y_1 and y_2 is the function meaningfully defined? Derive the demand functions.
   b. Interpret the demand functions.
   c. Determine whether the goods are complements or substitutes.

2. A consumer has a utility function $U(y_1, y_2) = -(a/y_1) - (b/y_2)$
   a. compute the demand functions for both y_1 and y_2 and determine their slopes.
   b. derive the Slustky equation and then show the substitution and income effects.
   c. compute the compensated demand functions for both y_1 and y_2 and determine their slopes.
   d. are the two goods complements or substitutes?
   e. what is the price elasticity of demand for both demand functions?
   f. derive the indirect utility function and then use Roy's Theorem to derive the demand function for y_1 and y_2.

3. Consider a consumer with the following utility function $U = y_1y_2$. Let his/her income and the following prices be M = \$240, p_1 = \$8, and p_2 = \$12.
   a. find the numerical solution for the consumer's utility maximizing consumption levels.
   b. derive the Slutsky equation for good 1 and determine its sign.
   c. are the two goods complements or substitutes?
   d. what is the price elasticity of demand for both demand functions?
   e. derive the indirect utility function and then use Roy's Theorem to derive the demand function for y_1 and y_2.
   f. what is the sign of $\partial y_2/\partial p_1$?

4. Why does the income effect upset the general law of demand? What is the relationship between the Engel curve and the income effect?

5. What does it mean when we say that two goods are substitutes, complements, and independents?

6. Geometrically determine whether the price elasticity of demand is elastic or inelastic given the following equation for a demand curve: $y = 500 - 4p_y$; and p_y = \$25.00.

7. Consider a consumer with a utility function of $U = y_1y_2$
   a. assume that M = \$20.00, p_1 = \$4.00, and p_2 = \$2.00, find the consumer equilibrium position.
   b. assume that M = \$20.00, p_1 = \$2.00, and p_2 = \$2.00, find the consumer equilibrium position.
   c. derive the consumer demand curve for y_1 and y_2.

d. derive the substitution and income effects for y_1 when M = \$20.00, p_2 = \$2.00, and p_1 = \$2.00 and \$4.00.
e. derive the price elasticity of demand for consumer demand curve y_1 when M = \$20.00, p_2 = \$2.00, and p_1 = \$2.00.
f. Are goods 1 and 2 substitutes, complements, or independents?

8. Describe the methodological procedure called comparative statics. What does this procedure imply about the nature of the consumer demand curve?

9. Consider the following utility function $U = 2y_1y_2$
   a. assume that M = \$20.00, p_1 = \$4.00, and p_2 = \$2.00, find the consumer equilibrium position.
   b. assume that M = \$20.00, p_1 = \$2.00, and p_2 = \$2.00, find the consumer equilibrium position.
   c. derive the consumer demand curve for y_1 and y_2.
   d. derive the substitution and income effects for y_1 when M = \$20.00, p_2 = \$2.00, and p_1 = \$2.00 and \$4.00.
   e. derive the price elasticity of demand for consumer demand curve y_1 when M = \$20.00, p_2 = \$2.00, and p_1 = \$2.00.
   f. Are goods 1 and 2 substitutes, complements, or independents?

10. Under what conditions can the individual consumer demand curves be aggregated to give a market demand curve that would behave as if it represented the decisions of a single maximizing consumer. What does this imply about the generality of neoclassical demand theory?

11. State the weak and strong axioms of revealed preference. How are the axioms related to the consumer's budget constrained utility maximization problem?

12. Give two ways to measure complementarity and substitutability of goods. Give an example in which the two measures contradict each other.

13. Discuss the compensated demand function. In your answer include:
    a. its definition.
    b. its properties.
    c. reasons for these properties.
    d. its relationship to the Slutsky equation if any.
    e. its possible usefulness in economic theory.

14. Discuss the expenditure function. In your answer include:
    a. its definition.
    b. its properties.
    c. reasons for these properties.
    d. its relation to the compensated demand curve.
    e. its possible usefulness in economic theory, particularly in the evaluation of compensating variation and equivalent variation.

15. Use the envelope theorem to derive the Slutsky equation for fixed money income y.

16. Consumer faces a strictly quasi-concave utility function $U = \mu(y_1,y_2)$ and a budget constraint of $p_1y_1 + p_2y_2 = M$.
    a. show the conditions for consumer equilibrium.
    b. prove that it is a utility maximization position.
    c. from the 1st order conditions derive the Slutsky equation for $\partial y_1/\partial p_1$ and identify which terms are the substitution effect and which are the income effect.
    d. from the 1st order conditions, identify and determine the sign of $\partial\lambda/\partial p_1$.
    e. using the above results, derive the indirect utility function and then use Roy's theorem to derive the demand function for y_2.
    f. using the above results, answer the following questions about substitutes and complements:
        (1) what is the difference between substitutes and complements?
        (2) what is the sign of $\partial y_2/\partial p_1$?
        (3) under what conditions can a good be a net substitute and a gross complement?

17. Revealed Preference
    a. Discuss each of the following statements and the relation between them.
        (1) If a consumer consumes the bundle y^1 at the prices p^1 and consumes y^2 at the prices p^2, and if $p^1y^1 > p^1y^2$, then the consumer is better off at the prices p^1.
        (2) If the aggregate consumption bundle in an economy is y^1 at prices p^1 and y^2 at prices p^2 then the society is better off at p^1 whenever $p^1y^1 > p^1y^2$.
    b. Is the following set of price-quantity data consistent with utility maximization? Why or why not?

        $p^1 = (1,2,3)$ $\quad$ $y^1 = (3,2,1)$
        $p^2 = (2,1,2)$ $\quad$ $y^2 = (2,2,1)$
        $p^3 = (3,5,1)$ $\quad$ $y^3 = (1,2,1)$

    c. Is the following set of price-quantity data consistent with utility maximization? Why or why not?

        $p^1 = (1,2,3)$ $\quad$ $y^1 = (3,5,3)$
        $p^2 = (2,1,3)$ $\quad$ $y^2 = (2,8,5)$
        $p^3 = (1,1,4)$ $\quad$ $y^3 = (4,7,3)$

    d. Show that strict convexity of indifference curves implies that the weak axiom of revealed preference is satisfied. What modified form of the weak axiom of revealed preference is implied by non-satiation (without strict convexity)?

18. Define and discuss the concept of utility tree.

19. What is new about Lancaster's "New Theory of Demand"? What aspects of conventional demand theory does Lancaster retain in his new theory? Lancaster's theory has not

gained wide acceptance among neoclassical economists. Why?

20. Given the following utility function $U = 2y_1y_2 + 4y_2$ answer the following questions:
    a. derive the marginal utility for y_2. What direction is it moving?
    b. derive the demand curve for y_2. What is its shape?
    c. calculate the income elasticity of demand for y_2. Is y_2 a normal, superior, or inferior good?
    d. using the Slutsky equation for y_2, derive the income and the substitution effect.
    e. determine whether y_1 and y_2 are gross complements or gross substitutes.
    f. derive the compensated demand curve for y_2.

21. To what degree was Hick's reformulation of consumer behavior and demand an advance over Marshall's theory of demand? Why is it possible to claim that neither Marshall's or the modern theory of demand are adequate?

22. What is the importance of duality for neoclassical demand theory?

23. Marshall's analysis of demand is not troubled by the Giffen good paradox while modern demand theory is. Why?

24. Is modern theory of consumer demand behavior and demand an advance over Marshall? Discuss.

25. Consider a consumer with the following utility function $U = y_1^{1/2}y_2^{1/2}$
    a. Derive the equilibrium demand functions for y_1 and y_2.
    b. Derive the Slutsky equation for $\partial y_1^e/\partial p_1$, identify which terms are the substitution effect and which are the income effect, determine the signs for the substitution and income effects, and determine the sign of $\partial y_1^e/\partial p_1$.
    c. Derive the indirect utility function and from it derive y_1^e.
    d. Derive the compensated demand functions for y_1 and y_2.
    e. What is the sign of $\partial y_2^u/\partial p_1$? Is it a net substitute, complement, or independent?
    f. Is the utility function a homogeneous function? Is it a homothetic function?

26. Separable utility functions weaken the substitution effect. Why?

Criticisms

1. Neoclassical theory of consumer behavior and demand has a number of theoretical problems. Examine one of them.

2. What is Steedman's criticism of neoclassical theory of consumer behavior? What implications does his criticism have for the neoclassical theory of demand?

3. Under what conditions can intransitive choices be made?

4. "In the tranquil view which neoclassical price theory presents there is but one dark spot which disturbs the harmony of the whole and, in fact, reduces the theoretical scope of the theory to virtually nothing at all. This is represented by the demand curve." Discuss.

5. The income effect has negative implications for Marshall's theory of demand and for modern demand theory. What are these negative implications and how did Marshall and modern neoclassical economists deal with them?

6. In what ways can the existence of radical uncertainty undermine the neoclassical utility function and demand curve?

7. Can preferences ever be asocially given?

8. Without marginal utility can there be an income effect, substitution effect, cross-effects, or a demand curve?

9. To what degree is the assumption of utility maximization a questionable assumption?

10. Income is troubling to both Marshall's demand theory and modern demand theory. Why?

11. If preferences are socially created, can a demand curve exist? Discuss.

12. Under what conditions can the individual consumer demand curves be aggregated to give a market demand curve that behaves as if it represents the decisions of a single consumer. What does this imply about the generality of neoclassical demand theory?

PROBLEM SET 3

Theory of Production and Costs

Marshall's Analysis of Supply

1. Marshall's discussion of the agents of production, production, and the laws of production

is logically faulty and confusing, but it is far superior to modern production theory. Discuss.

2. The law of diminishing marginal utility and the law of diminishing returns hold identical places in Marshall's theory of demand and analysis of production and costs; however, this is not the case for modern demand theory and theory of production and costs. Explain.

3. Marshall turned the agents of production, land, labor, and capital, into conceptually homogeneous categories. Why did he do it? What implications did this have for his discussion of returns to scale and the falling long period average total cost curve? Also state why modern production theory made no attempt to do the same thing?

4. Describe and contrast Marshall's conception of increasing returns to scale with that found in modern production theory.

5. What did Marshall mean by internal economies and external economies? Are they adequately captured in the modern production concept of returns to scale? Why or why not?

6. What is the difference between extensive rent and intensive rent? What is the relationship between intensive rent and marginal products?

7. What are prime costs, supplementary costs, expenses of production and real cost of production?

8. What is the difference between real cost of production and expenses of production? Why does Marshall believe it is important to make such a distinction?

9. Marshall linked specialized instrumental capital and specialization of labor to the scale of production. What problem did this cause him when discussing the laws of increasing returns?

Theory of Production

1. What properties must the production function have to ensure that it is a `proper' production function?

2. A production function, $f(x_1, x_2)$ is linearly homogeneous. Prove that an increasing average product of x_1 implies a decreasing marginal product of x_2.

3. What is the relationship between total output, average product, and marginal product? What are the stages of production and in which stage will production take place?

4. What is the relationship between the function coefficient and the stages of production?

5. What are the properties of an isoquant?

6. Suppose that a variable input in a production process is free (e.g. air). At the optimum, what is the numerical magnitude of the marginal product of that input?

7. Derive the marginal product and average product for input 1, the isoquant map, the function coefficient, and the elasticity of substitution from the following Cobb-Douglas production functions:

   a. $y = (x_1)^{.5}(x_2)^{.25}$
   b. $y = (x_1)^{.75}(x_2)^{.25}$
   c. $y = (x_1)(x_2)^{.25}$

8. Consider the following production schedule:

fixed input	50	50	50	50	50	50	50	50	50	50	50
variable input	0	1	2	3	4	5	6	7	8	9	10
output	0	20	50	75	95	110	120	125	125	120	115

   a. Compute the average and marginal product of the variable input.
   b. Does this schedule exhibit diminishing returns?
   c. Assuming input prices are constant, what would you predict about the AVC and ATC for this product?

9. Given the production function $y = L^{.5}K^{.5}$, where L stands for labor and K for capital, answer the following questions:

   a. Plot isoquants for output levels of 4 and 8.
   b. Show that these isoquants have a diminishing MRTS.
   c. Assuming the price of labor = price of capital = $10.00, derive the firm's expansion path and long period total cost curve.

10. Graph the following types of isoquants:

   a. To produce a cake, it always takes one stick of butter and three cups of flour.
   b. To produce a pound of wire it does not matter if you use copper or aluminum or both since they are both the same.

11. What is the elasticity of substitution? Given a Cobb-Douglas production function, calculate its elasticity of substitution.

12. Consider the following production function: $y = (x_1)^{.5} + x_2$

   a. Derive the marginal product for the first input and determine the direction of its movement.
   b. Derive the marginal product of the second input and determine the direction of its movement.

c. Show that the production function exhibits decreasing returns to scale.
d. Derive the marginal rate of technical substitution.
e. Fill in the following values for x_1 given the following values for y and x_2

y	x_2	x_1
0.5	0.5	
1.0	0.5	
2.0	0.5	
3.0	0.5	
4.0	0.5	
5.0	0.5	
6.0	0.5	

13. Is it possible for a production function to exhibit increasing returns to scale and diminishing marginal products simultaneously? Explain.

14. The two principle characteristics used to classify production functions are elasticity of substitution (denoted by σ) and returns to scale (denoted by RS). Given the following three production functions:

(1)	(2)	(3)
$\sigma = \infty$	$\sigma = 0$	$\sigma = 1$
RS (constant)	RS (decreasing)	RS (increasing)

a. First define σ and RS in general.
b. Draw an isoquant diagram to depict each of the three cases above.
c. State in explicit equation form an example of a production function for each case.

15. Prove that production can only take place where the production function is locally concave. Also prove that diminishing marginal products can be associated with increasing returns to scale whereas diminishing average products are only associated with decreasing returns to scale.

16. Define the law of variable proportions.

17. Designed to answer a particular set of questions, neoclassical theory of production abstracts from much of the descriptive detail of the process of production. What are the questions? Why do neoclassical economists want to answer them the way they do?

18. Given the following production function $y = 4x_1^{.5} + 2x_2^{.5}$ answer the following questions:

a. Derive the marginal product for x_1. What direction is it moving?
b. What is the relationship between total output, average product, and marginal product?
c. What is the returns to scale of the production function?

19. Given the following production function $y = 4(x_1^{.5})(x_2)$ answer the following questions:

a. Derive the marginal product for x_1. What direction is it moving?
b. What is the relationship between total output, average product, and marginal product?
c. What is the returns to scale of the production function?

20. Why will production not take place in stage one of production?

Theory of Costs

1. Using isocost lines and isoquants, derive the optimal input combination using the cost minimization approach when:

   a. Both inputs are variable.
   b. When only one input is variable.

2. What is the relationship between short period and long period costs?

3. What are the properties of the (constant output) factor input demand functions? What is a normal factor input? What is an inferior factor input? Using isoquants and isocost curves, show the situation in which input 1 is a normal factor input and which it is an inferior factor input.

4. Using isoquants and isocost lines, show graphically that an increase in the price of x_1 will result in a reduction in the quantity demanded of x_1.

5. A fixed plant is used to manufacture radio sets and, if Y sets are turned out per week, the variable costs are $\$(3Y + .5Y^2)$ and the marginal cost curve is $\$3 + Y$. Show that average variable costs increase steadily with output. If fixed costs are $100.00 a week, find the average total costs for a range of output and graph the average total cost curve. What is the minimum value of average total costs?

6. Consider the following production function $y = (x_1)^{.25} + (x_2)^2$
   a. Show that the production exhibits increasing returns to scale.
   b. What is the direction of movement of MP_1?
   c. Letting $p_1 = \$2.00$ and $p_2 = \$500.00$, derive the ATC and MC curves for levels of output 1 to 5, and prove that the firm's marginal cost curve slopes upward because of the law of diminishing returns.

7. Prove that the cost function C(y, p), which is defined as the minimum cost of producing the rate of output y at input prices, p, is concave in p.

8. In consumer theory, the slope of an individual's demand curve for a good is indeterminate. In production theory, the slope of a competitive firm's demand curve for an input is determinant. Explain.

9. Why are (constant output) factor input demand functions used to construct cost

functions?

10. Long run marginal cost must be higher than short tun marginal cost since the latter does not include the cost of capital. True or False. Explain.

11. Short Period--the Firm

<u>Data</u> variable costs: $VC = 4y^2 4y + 1$
fixed costs: $FC = \$2.00$
price of the first input = \$4.00
price of the second input = \$4.00

<u>Questions</u>

a. Derive the firm's marginal cost curve and determine its shape.
b. Show the relationship between the production function $y = (x_1)^{.5} + x_2$ and the total cost function $4y^2 - 4y + 3$. Use this relationship to show that the firm's marginal cost curve slopes upward because of the law of diminishing returns.

12. Given the following production function: $y = [(x_1)^{.5} + (x_2)^{.5}]^2$ answer the following questions:

a. Derive the marginal product for x_1 and what direction is it moving?
b. What returns to scale does the production exhibit?
c. Assume $x_2 = 3$, derive the firm's short period total cost function.
d. Derive the firm's short period marginal cost function and show that its shape depends on the law of diminishing returns.
e. Assume that $x_1 = 4$, $x_2 = 1$, $p_1 = \$9.00$, and $p_2 = \$9.00$, find y, ATC, AVC, and MC.
f. Define production function.

13. Given the following production function $y = .5(x_1)^{.5} + 4x_2$ answer the following questions:

a. Derive the marginal product for x_1 and what direction is it moving?
b. What returns to scale does the production exhibit?
c. Assume $x_2 = 2$, derive the firm's short period total cost function.
d. Derive the firm's short period marginal cost function and show that its shape depends on the law of diminishing returns.
e. Assume that $x_1 = 64$, $x_2 = 4$, $p_1 = \$5.00$, and $p_2 = \$10.00$, find y, ATC, AVC, and MC.

14. For the following total cost function $TC = 2y^3 – 3y^2 + 12y$, show that marginal costs cuts ATC at its lowest point.

15. Contrast production in the short period with production in the long period in relation to:

a. Expansion paths.
b. Cost curves.

16. Total Cost Functions

a. Why are constant output demand functions used to construct total cost functions?
b. Define total cost functions.
c. What is the difference between short period and long period cost functions?
d. Although mathematically $\partial\lambda/\partial y \geq< 0$ if we have a strictly quasi-concave production, why must its sign be positive?

17. What is the envelop theorem? What theoretical story does it attempt to tell? Critically evaluate this story.

18. Given the following production function $y = .25(x_1)^{.5}x_2$ answer the following questions:

a. Derive the marginal product for x_1. What direction is it moving?
b. Assume that $x_2 = 10$, derive the firm's short period total cost function, average total cost function, average variable cost function, and marginal cost function. At what level of output will minimum average variable cost and minimum average total cost occur?
c. Show that the shape of the marginal cost curve depends on the law of diminishing returns.
d. Assume that $x_1 = 400$, $x_2 = 10$, $p_1 = \$0.25$, and $p_2 = \$3.00$, find y, ATC, AVC, and MC.

19. What is the relationship between the function coefficient and long period average total costs and marginal costs?

20. Why is the duality of cost and production functions important for neoclassical economists?

21. What is the envelope theorem and what theoretical story does it attempt to tell? Critically evaluate this story.

22. Given the following production function $y = 4x_1^{.5} + 2x_2^{.5}$ answer the following questions:

a. Assume $x_2 = 16$, derive the constant output demand function for x_1 and the Lagrangian multiplier function.
b. Derive the short period total cost function in terms of y for prices $p_1 = \$5.00$ and $p_2 = \$3.00$.
c. Derive the marginal cost function and determine direction of movement as y increases.
d. Prove that marginal costs increase because the marginal product of x_1 declines.

e. Show that average variable costs increase as y increases. At what level of output are marginal costs and average variable costs equal.
f. What is the relationship between the function coefficient and long period average total costs and long period marginal costs?

23. Given the following production function $y = 4(x_1^{.5})(x_2)$ answer the following questions:

a. Assume $x_2 = 10$, derive the factor input demand function for x_1 and the Lagrangian multiplier function.
b. Derive the short period total cost function in terms of y for prices $p_1 = \$3.00$ and $p_2 = \$5.00$.
c. Derive the marginal cost function and determine direction of movement as y increases.
d. Prove that marginal costs increase because the marginal product of x_1 declines.
e. Show that average variable costs increase as y increases.
f. At what level of output and costs are marginal costs and average total costs equal?

Special Topics in Production and Cost Theory

1. The Cobb-Douglas production function, defined by $y = \varphi x_1^{\alpha} x_2^{\beta}$ where φ, α, and β are positive parameters, is commonly employed in empirical economics.

a. Show that this production function is homogeneous. What is the degree of homogeneity?
b. What constraints on the parameters would make the production function homogeneous of degree one?
c. Show that the marginal rate of technical substitution can be written as a function of the input ratio: $MRTS_{12}(x_1, x_2) = \phi(x_1/x_2)$.
d. Show that this production function is homothetic (for any positive values of the parameters).
e. Show that the expansion path is a straight line.
f. Derive the cost function (assuming that $\alpha + \beta < 1$).
g. Derive the output-constrained input demand functions.
h. What constraints on the parameters result in returns to scale being equal to returns to outlay?

2. The Constant Elasticity of Substitution is commonly employed in empirical studies. It is defined as $y = \varphi[\alpha x_1^{\beta} + (1 - \alpha)x_2^{\beta}]^{1/\beta}$ where φ and α are positive parameters and $0 < \beta \leq 1$.

a. Show that this function is homogeneous of degree one.
b. Show that the expansion path is a straight line.
c. Show that $MRTS_{12}(x_1, x_2) = \phi(x_1/x_2)$.
d. Show that the elasticity of substitution is constant.

3. Show rigorously from the duality properties of the cost function the relationship between

a factor's output elasticity and the response of marginal cost to a change in the factor's price. What are four uses of the dual relationship between the description of the production function and its associated cost function?

4. You are given the cost function: $c(p_1, p_2, y) = yp_1^{\alpha} p_2^{1-\alpha}$

   a. Derive the production function.
   b. What assumptions did you make in part (a)?

5. You are given the cost function: $c(p_1, p_2, y) = p_2[1 + y + \log(p_1/p_2)]$

   a. Derive the production function.
   b. What assumptions did you make in part (a)?

6. What is Shepard's Lemma?

7. Given the following total cost function: $TC = 2yp_1^{.5}p_2^{.5}$:

   a. Derive x_1^e, the factor input demand function for x_1.
   b. Derive x_2^e, the factor input demand function for x_2.
   c. How does x_1^e react when its own price is reduced?
   d. Are x_1^e and x_2^e complements or substitutes?
   e. Is x_1^e a normal or inferior factor input?
   f. Derive the production function from the total cost function.

Criticism

1. The neoclassical theory of production provides a model which abstracts from much of the descriptive detail of the processes of production and is designed to answer a particular set of questions. Succinctly outline the neoclassical theory of production, and then answer the following.

   a. Why would a neoclassical theorist view abstraction in the theory of production as desirable, and why might others view the loss of descriptive detail less favorably?
   b. Why might some critics of the neoclassical view of production favor theoretical abstraction in general but prefer a different abstract view of production in order to answer a different set of questions?

2. Discuss the problems that emerge when trying to make the law of diminishing returns a technical relationship.

3. Modern production and cost theory says that short period marginal cost curves slope upward and long period average total cost curves are U-shape; however, empirical evidence seems to show otherwise. Is the theory daft, evidence wrong, or economists have no idea what they are doing? Explain.

4. What are the implications of Yordon's analysis of the short period cost function in manufacturing for neoclassical production and cost theory?

5. Why is it problematical to assume that technology is given asocially and is given in such a way as to be appropriate for neoclassical theorizing?

6. Is it possible for firms to adopt technology in which the output is maximized by the given inputs?

7. How do factor input demand functions violate the *ceteris paribus* assumption? What does this imply about the neoclassical theory of costs?

8. What problems does a socially-based production function pose for the concept of relative scarcity? Do factor inputs need to be scarce?

9. Under what condition would a production function not generate a marginal product? And what impact would this have for the relationship between output and costs?

10. Why is it problematical to assume that technology which is socially constructed is appropriate for neoclassical theorizing?

PROBLEM SET 4

Price Theory: Perfect Competition

Marshall's Theory of Prices

1. State those aspects of Marshall's short-period theory of prices that are consistent with perfect competition; and state those aspects which are not consistent with perfect competition.

2. Discuss Marshall's problem with increasing returns to scale. Explain how Marshall combined increasing returns to scale and competitive conditions in his long-period theory of prices.

3. What role does Marshall's time periods play in his theory of value?

4. Why did Marshall introduce the firm into his theory of prices?

5. Marshall's concept of free competition is realistic but logically flawed; on the other hand, the modern concept of perfect competition is logically consistent but unrealistic. Discuss.

6. Describe Marshall's representative firm and explain its role in his theory of prices.

The Years of Turmoil, 1920 – 1933

1. Describe Sraffa's critique of the law of diminishing returns.

2. What were Sraffa's reasons for concluding that it was not possible to have a downward sloping market supply curve?

3. What were Sraffa's reasons for concluding that it was not possible to have an upward sloping market supply curve?

4. Compare Marshall's representative firm with Pigou's equilibrium firm.

5. In his 1926 article, Sraffa makes the following statement:

> It is necessary...to abandon the path of free competition and turn in the opposite direction...towards monopoly. Here we find a well-defined theory in which variations of cost connected with changes in the dimensions of the individual undertaking play an important part.

   a. Why did Sraffa argue that `free competition' must be abandoned if one wanted a demand and cost (supply) based long-period theory of prices?

   b. Describe the long-period theory of prices Sraffa outlined in the latter part of his 1926 article.

6. What was Sraffa's critique of Marshall's theory of prices and how did the critique lead to the emergence of the theory of imperfect competition?

7. In his article "On the Relation Between Cost and Quantity Produced," Sraffa argued that free competition must be abandoned if one wanted a demand and cost (supply) based long period theory of prices. Why?

8. What is Marshall's explanation for stable short-period prices and why is it inconsistent with neoclassical price theory?

9. Consider the following quote by P. Sraffa from the "Increasing Returns and the Representative Firm" Symposium:

> I am trying to find what are the assumptions implicit in Marshall's theory; if Mr. Robertson regards them as extremely unreal, I sympathise with him. We seem to be agreed that the theory cannot be interpreted in a way which makes it logically self-consistent and, at the same time reconciles it with the facts it sets out to explain. Mr. Robertson's remedy is to discard mathematics, and he suggests that my remedy is to discard the facts; perhaps I ought to have explained that, in the circumstances, I think it is Marshall's theory that should be discarded.

Why does Sraffa suggests that Marshall's theory should be discarded?

Perfect Competition and the Supply Curve

1. Consider a profit-maximizing firm that employs a single variable input. Suppose that the output, y, for each level of x is given by the following schedule:

y	1	2	7	10	12	13	13	12	10
x	0	1	2	3	4	5	6	7	8

   a. Compute the schedules for the average and marginal product.
   b. Let $p_y = 2$ and $p_x = 4$, determine the level of output that would maximize the firm's profits. How much of the input is purchased by the firm at this level of output?

2. The total cost schedule for a profit-maximizing firm is as follows:

y	0	1	2	3	4	5	6	7	8	9	10	11
TC	10	19	26	32	37	41	45	50	56	63	72	83

   a. Derive the schedules for total costs, average total costs, variable costs, average variable costs, average fixed costs, and marginal costs.
   b. Let $p_y = \$7.00$, determine the profit maximizing level of output.

3. Consider a firm facing the following total cost function: $TC = .5y^2 + 10y + 5000$:

   a. If $p_y = \$100.00$, find the profit maximizing output and determine the amount of profits the firm made.
   b. Would this firm continue to operate in the short period? In the long period?

4. Prove that the firm's short period marginal cost curve is its short period supply curve. Under what conditions will the firm not supply a good in the short period?

5. Prove that the firm's long period marginal cost curve is its long period supply curve. Under what conditions will the firm not supply a good in the long period?

6. Explain how the short period market supply curve is derived and provide an explanation

for its shape.

7. A perfectly competitive industry consists of n firms, each with identical marginal costs curves of the form $MC_i = 1 + 4y_i$ where y_i is the output of the ith firm:

   a. Derive the industry supply curve.
   b. Find the market clearing price and quantity if the market demand curve is $p_y = a - by$ where $a,b > 0$.

8. What are the assumptions of perfect competition?

9. Describe the Walrasian procedure for the determination of the short period market price and output and explain why this procedure produces a stable equilibrium. Relate your answer to Marshall's approach for determining market price and output.

10. Why is the perfectly competitive long period theory of prices considered to be nonexistent?

11. A perfectly competitive market is comprised of 1000 producers each with a variable cost curve defined by $VC = 20y + .5y^2$. The market demand function is defined by $y_{md} = 105{,}000 - 500p_y$.

    a. Derive the short period supply curve of a producer;
    b. Derive the short period market supply curve;
    c. What is the equilibrium price and quantity for the market;
    d. What is the output for each producer; and
    e. If each producer has fixed costs of $200, what is the profit of each producer?

12. Find the profit-maximizing quantity of corn for the firm, given the following cost data:

quantity	0	10	20	30	40	50	60	70	80
total cost	100	140	170	190	200	210	230	260	300

price of corn = $3.00 per bushel

13. Analyze the effect on the equilibrium price and quantity in a perfectly competitive market if the price of a variable input increases. Draw the appropriate diagrams.

14. How is it possible to have an equilibrium in the short period when a typical firm is earning profits that may be greater than or less than zero?

15. A firm in a perfectly competitive market has cost functions of the form $TC = 5000 - 10y + y^2$:

    a. Find the profit-maximizing quantity for the firm if the market price p_y is $200;
    b. Calculate the profits; and
    c. Derive the short period supply curve for the firm.

16. Working in the short period, assuming the following cost curves for the competitive firm: variable cost curve = $7y^2$ and fixed costs = $50.00, answer the following questions:

    a. Assume that the market price is $98.00, determine how much the firm will produce in order to maximize its profits and determine its economic profits.
    b. Derive the firm's supply curve.
    c. Under what conditions will the firm not produce in this problem; under what conditions will it produce but make zero economic profits.

17. Answer the following questions: all these questions deal with the market in the short period.

    Data

    consumer A demand curve: $y_d = 20 - 2p_y$
    consumer B demand curve: $y_d = 25 - 3p_y$
    consumer C demand curve: $y_d = 19 - p_y$
    firm A supply curve: $y_s = 5p_y$
    firm B supply curve: $y_s = 3p_y$
    firm C supply curve: $y_s = 2p_y$

    Questions

    a. If the auctioneer calls out a market price of $6, how much would each consumer demand in the market and how much would each firm supply to the market. Does excess demand or excess supply exist in the market?
    b. If the auctioneer called out a market price of $2, how much would each consumer demand in the market and how much would each firm supply to the market. Does excess demand or excess supply exist in the market?
    c. Derive the market demand curve and the market supply curve.
    d. Determine the equilibrium market price and quantity. How much will each consumer demand and each firm supply at the equilibrium market price?
    e. Why does this auctioneer procedure produce a stable equilibrium?

18. Answer the following questions: all these questions deal with the long period.

    Data

    firm A: $ATC = y^2 - 2y + 8$
    firm B: $ATC = y^2 - 2y + 6$
    firm C: $ATC = y^2 - 2y + 5$
    market demand curve: $y_m = 225 - 15p_y$

    Questions

a. Assume that the minimum ATC for each of the firms occurs when they produce one unit of output. If the auctioneer calls out a market price of $6, which firm will have to leave the market and why?

b. Determine the long period equilibrium market price, equilibrium market output, and the number of firms in the market.

c. Let us now assume that Dr. Bell has developed a new way of producing the good in question and his ATC = 363 - 2y. Using your answers from (b), if Dr. Bell enters the market at the existing equilibrium price, how much would be supplied for the market? Why would the market not be in equilibrium? Is there a market price at which the quantity demanded would equal the quantity supplied? Why is it not possible to derive a supply curve for Dr. Bell's firm or a supply curve for the market in this situation?

19. Answer the following questions: all of these questions deal with the long period.

a. Assume that firm A has the following total cost curve: $TC = 100y - 2y^2$

(i) If the market price is $30, how much will firm A produce and what will be its economic profits?

(ii) Is it possible for the above marginal cost curve to be the firm's supply curve?

b. Assume that firm B has the following total cost curve: $TC = 50y$

(i) If the market price is $50, how much will the firm produce in order to maximize its profits?

(ii) Is it possible for firm B to have a supply curve, given these cost conditions? Why or why not?

20. Solve the following system of equations for p_y and y_m

a. $y_{md} = 18 - 2p_y$
$y_{ms} = -6 + p_y$

b. $y_{md} = 15 - 4p_y$
$y_{ms} = 6p_y - 1$

21. Let the market supply curve be $y_{ms} = 3p_y + 12$

Let the firm supply curve be $y_{fs} = (1/8)p_y + .5$

Let the market demand curve be $y_{md} = 336 - 6p_y$

Determine the equilibrium market quantity, the equilibrium market price, the equilibrium quantity the firm will supply to the market, and the number of firms in the market.

22. Describe the Edgeworth recontracting procedure for the determination of the short period market price and explain why this procedure produces a stable equilibrium.

23. Assume that the firm has the following long period ATC: $ATC = 10 - .5y^{.5}$

    a. Determine the shape of the ATC curve.
    b. If the market price is $8.00, how much will the firm supply in order to maximize its profit?
    c. Is it possible to derive a firm supply curve, given the firm's ATC curve? Why or why not?

24. Can market equilibrium exist if individual firms are making positive economic profits in the long period?

25. If a firm has the following ATC curve $ATC = y^2$, what must the market price be for the market to be in equilibrium in the long period and how much will the firm produce in this case?

26. Assume a perfectly competitive market that is in long period equilibrium. Now assume that market demand has declined. In the new long-period equilibrium what can we say about price and output of each firm and for the market as a whole?

27. Long Period

    Data

    Firm A: $TC = 4y^3 - 8y^2 + 8y$
    Firm B: $TC = 5y^3 - 10y^2 + 6y$
    Firm C: $TC = 6y^3 - 12y^2 + 9y$
    Market Demand Curve: $y_m = 310 - 25p$

    Questions

    a. Assume that the market price is $5.00. Find the quantity of output Firm A would produce in order to maximize its profits and calculate its economic profits.
    b. Repeat question (a), but assume that the market price is now $3.00.
    c. Determine the long period equilibrium market price, equilibrium market output, and the number of firms in the market.
    d. Repeat question (c), but the market demand curve has now shifted to $y_m = 340 - 5p$.
    e. Relate the results of (c) and (d) to the viability of the long period supply and demand theory of prices.
    f. Assume that Firm B has adopted a new technique of production that gives it the following total cost structure: $TC = 1178y - 5y^2$
        (i) Assuming that the market price is $3.00, how much will Firm B produce in order to maximize its profits?
        (ii) Assuming the initially given market demand curve, derive the profit maximizing price and quantity for the firm.

28. Using calculus show how to derive the firm's short period supply function and long period supply function.

29. Supply Curve

    a. Prove that marginal costs equals the supply price.
    b. Prove that the marginal cost curve is the supply curve.
    c. Under what conditions will a firm not supply a good, given positive prices.

30. Given the following production function $y = K^{\alpha}L^{1-\alpha}$ for a perfectly competitive firm derive the short period profit maximizing output (assuming K is fixed) using the Walrasian approach.

31. A short period perfectly competitive market consists of 42 firms, each with a total cost function of $TC = 7y^2 + 63$. The market demand function is $y_{md} = 420 - 7p_y$.

    a. If the market price is $70.00, how much will the firm produce and what will be its economic profits?
    b. At what market price will the firm make zero economic profits?
    c. Derive the firm supply curve.
    d. Derive the market supply curve.
    e. Determine the market equilibrium price, market equilibrium quantity, how much will each firm produce, and what will be the firm's economic profits.

32. Assume that three different firm cost structures exist in a competitive market and that the minimum ATC for each cost structure occurs when output is equal to 3:

    Firm cost structure 1: $ATC = y^2 - 3y + 10$
    Firm cost structure 2: $ATC = y^2 - 3y + 15$
    Firm cost structure 3: $ATC = y^2 - 3y + 20$

    a. Given the following cost structures, determine the long period market price.
    b. Assume the market demand function is $y_{md} = 490 - 7p_y$, what will the quantity supplied to the market and how many firms will be in the market?
    c. Assume that the market demand curve shifts to $y_{md} = 810 - 9p_y$, what is the new long period market price and market quantity?
    d. What are the implications of these results of a competitive long period theory of prices?

33. A short period perfectly competitive market consists of 60 firms, each with a total cost function of $TC = 10y^2 + 80$. The market demand function is $y_{md} = 600 - 7p_y$.

    a. If the market price is $80.00, how much will the individual firm produce and what will be its economic profits?
    b. Derive the market supply curve.
    c. Determine the market equilibrium price, market equilibrium quantity, how much

will each firm produce, and what will be the firm's economic profits.

34. Assume that a perfectly competitive firm has the following **long period** total cost curve: $TC = 500y - 5y^2$.

    a. For a given market price, $p_y = \$11.21$, prove that the enterprise will not produce where marginal cost equals the market price.
    b. Why is it not possible to derive a supply curve for the firm in this situation?

35. What are the two principle explanations of how the market price is determined in perfect competition? Why are they utilized?

36. When dealing with long period equilibrium under perfect competition, the combination of firms with increasing average total costs and free entry creates rather paradoxical results. What are the paradoxical results? How do neoclassical economists try to get around the results? And are they successful?

Criticisms

1. Critically evaluate the following statement:

    Assume that the output of a competitive market increases and as a result its input prices increase, then the outcome will be an increasing market supply curve. Consequently it is now possible for supply and demand to jointly determine the market price and quantity.

2. What impact does the absence of the supply curve outside of perfect competition have on the price mechanism for the allocation of scarce resources in a laissez-faire economy?

PROBLEM SET 5

Price Theory: Imperfect Competition and the Firm

Monopoly

1. A recession causes a monopolist's demand curve to shift to the left. Is it conceivable that the monopolist's profit- maximizing reaction will be to raise its price? If so, under what circumstances (assuming no increases in input prices)? Are there any plausible circumstances under which a competitive industry's price will rise in response to a recession-induced leftward demand curve shift?

2. A profit-maximizing monopolist never produces an output at which the price elasticity of demand is less than one (in absolute value). Why?

3. A supply curve is defined as a one-to-one relationship between price and quantity (associated with each price is a unique quantity supplied). Use a diagram to illustrate that it makes no sense to speak of the `supply curve of a monopolist'.

4. List the assumptions of the monopoly model.

5. What is the significance of the assumption that entry is blocked in monopoly?

6. Explain why the marginal revenue curve for a monopolist lies below the demand curve; that is, why is marginal revenue less than price at each positive quantity?

7. Find the profit-maximizing price and output for the monopoly, given the following information. Calculate the profits.

quantity	0	50	100	150	200	250	300	350
total cost	100	150	210	280	360	450	550	660
total revenue	0	150	275	375	450	500	525	525

8. Comment on the following statement: monopolists will always make a profit because they can charge any price they want for their product.

9. Construct a marginal cost schedule for a multi-plant monopolist from the information given. Find the quantity that will maximize profits if the firm produces. What quantity should be produced by each plant?

quantity	1	2	3	4	5	6	7	8	9	10
MC of plant A	15	16	17	19	21	24	28	33	39	46
MC of plant B	2	6	11	17	24	32	41	51	62	74
MR of the firm	44	42	40	38	36	34	32	30	28	26

10. The XYZ firm is a monopolist selling gizmos in two different markets. The demand curves for markets A and B are:

market A: $y = 200 - 10p_y$
market B: $y = 100 - 10p_y$

The marginal cost of production for the XYZ firm is $5.00 per gizmo. Find the profit-maximizing prices and quantities for each market.

11. Assume that the total cost curve for the firm is given by $TC = 52 - 2y + 3y^2$ and that the firm's demand curve is given by $y = 5 - .1p_y$. Find the profit-maximizing price and quantity for the firm. Calculate the profits.

12. Given the following market demand curve $p_y = 37 - 3y$ and the monopolist's total cost curve $TC = 4 + 2y^2$, answer the following questions:

    a. Determine the monopolist's profit maximizing price and quantity.
    b. Determine the monopolist's economic profits.
    c. Assume that the monopolist's demand curve has shifted and has taken the following form $p_y = 28 - .8y$

        (i) Determine the monopolist's new profit maximizing price, quantity, and economic profit.
        (ii) What do these results imply about the existence of a firm supply curve

under the conditions of monopoly and why?

13. Discuss the following quote:

"...[I]t has to be recognized that a general abandonment of the assumption of perfect competition, a universal adoption of the assumption of monopoly, must have very destructive consequences for economic theory." (Hicks, Value and Capital)

14. If a monopolist sells in two separate markets with different demand curves, in order to maximize its profits, it must charge a lower price where the elasticity of demand is lower in absolute value. Comment.

15. If the demand curve facing the monopolist is $y = 60 - p_y$, what is the monopolist's marginal revenue function?

16. If the demand curve facing the monopolist is $y = 120 - p_y$ and its cost curve is $TC = 2y^2 + 6y + 216$, find the following:

    a. Price, quantity and economic profits.
    b. The monopolist's minimum average total costs.

17. If a monopolist sells in two separate markets with different demand curves, in order to maximize its profits, it must charge a lower price where the elasticity of demand is lower in absolute value. Comment.

18. An industry consists of n firms, each with identical average total costs (ATC) functions of the form $ATC_I = (1/y_i) + y_i$ where y_i is the output of the ith firm.

    a. Find the marginal cost schedule of the ith firm.
    b. Derive the industry supply curve and find the market clearing price and quantity if the demand curve is $p_y = a - by$ where $a,b > 0$.
    c. Rederive equilibrium p_y and y assuming that the firms merge and operate a monopoly.

19. A profit maximizing monopoly discovers that it can increase demand by advertising. Units of advertising can be purchased as a price p_a.

    a. If total cost for the monopoly is $C = C(y,A)$ and demand for the monopoly is $y = y(p_y, A)$, A are the units of advertising, find the marginal conditions for optimal p_y and optimal A.
    b. Solve for optimal p_y and optimal A if

       $C = cy^2 + p_aA$ $\quad$ $c > 0$
       $Y = a - bp_y + f(A)$ $\quad$ $a, b > 0$
       $F(A) = e_1A + e_2A^2$ $\quad$ $e_1, e_2 > 0$

    c. Discuss whether an increase in the efficiency of advertising lowers or raises the monopoly price.

20. Data

Firm: $.5y^2 + 8y + 15$

Questions

a. Assume that the monopolist faces the following market demand curve $p = 23 - y$. Find its profit maximizing price and quantity. Also calculate its economic profits and the equilibrium price elasticity of demand.
b. Assume that the monopolist's demand curve has shifted and that in the new equilibrium position price elasticity of demand is 8.5 and $y = 7$; determine the new profit maximizing price.
c. Compare the results of (a) and (b) and state the appropriate conclusion.
d. Define the Lerner index of monopoly and calculate its value for the monopolist using the results from (a).

Monopolistic/Imperfect Competition

1. Why does the supply curve disappear in imperfect competition?

2. In imperfect competition, each firm has its own demand curve. Describe the basis for the demand curve and whether it can be considered a neoclassical demand curve (hint: Use Pigou's formulation of Robinson's firm demand curve to answer the question).

3. Describe and compare Chamberlin's long period analysis of price under monopolistic competition to Joan Robinson's long period analysis of price under imperfect competition.

4. The long period firm demand curve is theoretically problematical because of the problem of interdependency. How did Robinson and Chamberlin avoid the problem? What was Kaldor's criticism of Robinson's firm demand curve? Relate Kaldor's criticism to Sweezy's kinked demand curve.

5. In what ways is the long period imperfectly competitive theory of prices different from the long period perfectly competitive theory of prices?

6. What are the properties of the imperfectly competitive theory of prices?

7. Consider an imperfectly competitive firm with the following schedule of marginal costs:

quantity produced (supplied)	0	1	2	3	4	5	6
marginal costs	0	1	3	5	7	9	11

a. If marginal costs is \$3.00 and price elasticity of demand is 3, then determine the price the firm will set in order to maximize its economic profits.

b. If marginal costs is $5.00 and the price elasticity of demand is 6, then determine the price the firm will set in order to maximize its economic profits.
c. If marginal costs is $5.00 and price elasticity of demand is 5, then determine the price the firm will set in order to maximize its economic profits.
d. Taking the above results together, what do they imply about the existence of a firm supply curve under imperfect competition.

8. Assume that profits are negative in a imperfectly competitive industry in short period equilibrium. Explain the adjustment process by which profits will be driven to zero in long period equilibrium. Draw appropriate diagrams.

9. What aspects of Marshall's theory of prices were captured by the theory of imperfect competition?

10. The demand function for a firm in imperfect competition is given by $p_y = 30 - .75y$ and the average total cost function takes the form of $ATC = (30/y) + .3y$. Find the value for y which

a. Maximizes total revenue.
b. Minimizes average total costs.
c. Maximizes economic profits.

11. What criticisms does the Chicago School lodge against monopolistic competition?

12. Two firms are 10 miles apart, the price of their produce at works is the same for each and transport cost per mile per unit of the good is three times as high for one firm as for the other. Show that the former firm supplies an area within a circle of radius 3-3/4 miles. Draw a graph to illustrate the distribution of the market.

13. Assume that the total cost curve for a imperfectly competitive firm is given by $TC = 2y^2 + 25$. Also suppose it faces a demand curve of $p_y = 160 - 2y$.

a. Determine the firm's profit maximizing price and quantity and its economic profits.
b. Determine the price elasticity of demand at the profit maximizing price.
c. Assume that firm's demand curve has shifted and in the new equilibrium position the price elasticity of demand is 6 and the profit maximizing quantity produced is 25, what dose imply about the existence of a firm supply curve under imperfect competition and why?

14. Under imperfect/monopolistic competition as well as in Marshall's theory of prices, heterogenous firms make the existence of market equilibrium implausible. Why? How did Marshall, Robinson, and Chamberlin overcome this problem? Evaluate their solutions.

Oligopoly

Early Models of Competition Among a Few

1. Discuss joint-profit maximization with respect to cartels. Why is the theory of joint-profit maximization not a satisfactory theory of oligopolistic behavior?

2. Show that the equilibrium industry output for a Cournot duopoly model is two-thirds the output for a competitive industry with the same demand and cost conditions.

3. Explain why (a) significant entry barriers, (b) inelastic demand, and (c) homogeneous products facilitate the formation of a cartel.

4. The widget industry comprises 200 identical firms. The long period marginal cost curve, long period ATC curve, and long period market supply curve are represented by a horizontal line at $50. The industry demand curve is given by the equation $y = 5000 - 10p_y$. Find the equilibrium price and quantity for the industry if it behaves in a perfectly competitive fashion. Then assume that the 200 firms form a cartel and behave as a monopoly. Find the equilibrium price and quantity for the cartel. How would you set output quotas for the firms once the cartel is formed? What problems are you likely to encounter?

5. "It always pays firms to form a cartel; but once the cartel is formed, it always pays individual firms to cheat on the output quotas'. Under what conditions are either or both parts of the statement true?

6. Compare Cournot's duopoly model with Bertrand's dupoly model.

7. Cournot's duopoly model

   Data

   market demand curve $P = 100 - .4(Y_1 + Y_2)$
   total cost curve of firm 1 $TC_1 = 6Y_1$
   total cost curve of firm 2 $TC_2 = .7(Y_2)^2$

   Questions

   a. Derive the reaction curves for both firms and graph them.
   b. Solve for the market price and output.
   c. What will be the profit maximizing output for each firm and what will be their profits?

8. Describe Stackelberg's duopoly model.

9. Using the data in problem 7 above, answer the following questions:

a. Find the Stackelberg's solution with firm 1 being the sophisticated leader.
b. Find the Stackelberg's solution with firm 2 being the sophisticated leader.
c. What happens if both firms want to be the sophisticated leader?
d. What happens if neither firm wants to be the sophisticated leader?

Oligopoly

1. Firm demand curves become problematical under conditions of imperfect-oligopolistic competition where firm interdependency exists. How did economists, such as Joan Robinson, Edward Chamberlin, Robert Hall-Charles Hitch, and others, overcome this problem so as to be able to construct theoretical models and analysis of imperfect-oligopolistic competition.

2. Explain why there are several models or theories of oligopoly rather than just one.

3. Discuss Hall and Hitch's article, "Price Theory and Business Behaviour," and include in your discussion the following points: (i) a description of full cost pricing, (ii) the relationship between full cost prices and the kinked demand curve, (iii) their explanation of stable prices, and (iv) why they saw full cost pricing as an attack on the 'marginalist' explanation of the price setting behavior of businessmen.

4. Widgets are produced in an oligopolistic industry in which firms perceive their demand curves as kinked at the current price. Compare the price stability of the oligopoly with that of a monopolized industry.

5. In what ways are the Sweezy and the Hall and Hitch kinked demand curves different?

6. Delineate and compare Cournot's duopoly model, Chamberlin's duopoly model, and Hall and Hitch's kinked demand curve model. Which model seems to explain 'competition among the few' the best? Explain your answer.

7. Limit pricing model

   a. Compare Bain's limit pricing model with Sylos'Labini's limit pricing model.
   b. What were Modigliani and Bhagwati's contributions to the limit pricing model?

8. Are Bain's barriers to entry really barriers to existing large industrial corporations? Explain.

9. Structure-conduct-performance paradigm

   a. What is the welfare basis of this paradigm.
   b. How did E. S. Mason make Chamberlin's analysis 'operational'?
   c. What was J. S. Bain's contribution to the paradigm?
   d. Discuss the administered prices-concentration controversy.

e. Discuss the administered inflation-concentration controversy.
f. Discuss the profit-concentration controversy.

10. Assume that the firm has the following total cost curve and faces the following kinked demand curve:

$TC = 2y^2 + 6y + 10$
dd curve: $p_y = 60 - 2y$
DD curve: $p_y = 120 - 12y$

a. Determine the firm's equilibrium price and quantity and show that marginal costs does not equal marginal revenue.
b. What implications does this have for neoclassical price theory?

11. Discuss the following neoclassical models of price leadership:

a. Dominant firm price leadership model.
b. Barometric price leadership model.
c. Collusive price leadership model.

12. ABC Limited is much larger than its rivals in the gizmo industry. The industry demand curve is given by the equation $y = 1000 - 2p_y$. The marginal cost curve for all of the fringe firms in the industry is given by the equation $y = -50 + p_y$. The marginal and average total cost for ABC Limited is $10 per unit. Find the industry price and quantity, quantity supplied by the fringe firms, and quantity supplied by the dominant firm.

13. <u>Data</u>

Firm: $TC = 20 + 10y + y^2$

<u>Questions</u>

a. Assume that the firm faces the following demand curve: $p_y = 58 - 5y$. Find its profit maximizing price and quantity. Also calculate its economic profits and the equilibrium price elasticity of demand.
b. Assume the firm's demand curve has shifted and has taken the following form $p_y = 46 - y$:

   (i) Determine the firm's new profit maximizing price, quantity, and economic profit, and calculate the equilibrium price elasticity of demand.
   (ii) Compare the above results and discuss what they imply about the existence of a firm supply curve under the conditions of imperfect competition.

c. Assume the firm to be a dominant firm and that the market demand curve is $p_y = 75 - 5y$; also assume that the marginal cost curve of the fringe firm is $MC = 3y$:

(i) Derive the dominant firm's demand curve.
(ii) Determine the dominate firm's profit maximizing price and quantity.
(iii) Determine the profit maximizing quantity supplied by the fringe firm.

d. Assume that the firm faces the following kinked demand curve
dd curve: $p_y = 46 - y$
DD curve: $p_y = 73 - 10y$

(i) Graph the kinked demand curve and its marginal revenue curve.
(ii) Determine the firm's equilibrium price and quantity.
(iii) Show that marginal cost does not equal marginal revenue in equilibrium.

Behavioral and Managerial Theories of the Firm

1. In developing a theory of the firm, microeconomic theorists traditionally relied upon an assumption that firms seek to maximize profits. More recently, some theorists have argued that the owners and/or managers of private enterprises are fundamentally concerned with other objectives for their firms. What are some of the other objectives? Discuss how these other objectives might affect profitability.

2. As graduate students, economists are presented with many different theories of the firm. However, contrary to what is taught in demand theory, economists do not accept the axioms of continuity or transitivity when it comes to their preference ordering of alternative theories. Rather they prefer an ordering of theories--one theory must be `right' and the rest wrong to a lesser or greater extent. Assuming you are such an economist investigating a firm in an oligopolistic industry, what data would you need to collect and how would you use it to differentiate and rank the following theories of the firm: Baumol's sales maximizing theory of the firm, Williamson's managerial discretion theory of the firm, and Cyert and March's behavioral theory of the firm?

3. Baumol's sales maximizing theory of the firm

   a. Set up the model and derive the comparative static results with respect to price and output. Compare these results to the results derived from the profit maximizing theory of the firm.
   b. How does advertising affect price and output?

4. Discuss Marris's managerial theory of the firm.

5. Behavioral theory of the firm: what is it and why is it considered an alternative to profit maximization. In what ways is it different from the managerial theories of the firm?

6. What role does the separation of ownership from control play in the managerial theories of the firm?

7. Compare the goals of the firm for perfect competition and monopoly with the goals of the

firm found in the managerial and behavioral theories of the firm. How do the different goals affect the reaction of the firm when it faces a shift in demand or costs?

8. Discuss alternative theories of firm behavior. In your discussion give a coherent explanation of why a rational farmer might choose to apply so much pesticide as to be in the third stage of production.

9. Compare the predictive content of Baumol's sales maximizing theory of the firm with Williamson's managerial discretion theory of the firm.

Game Theory

[to be written]

Problems with Marginalism

1. Under what conditions can imperfect competition `explain' stable prices? Can this explanation be considered general enough to account for the existence of stable prices in the real world?

2. The consumer demand curve and the firm demand curve under imperfect competition are both questionable concepts. Delineated the criticisms and then say whether they can be overcome and the demand curve saved.

3. Discuss the following statement: marginalism is not designed to serve to explain and predict the behavior of real firms; instead, it is designed to explain and predict changes in observed prices as effects of particular changes in conditions. In this causal connection the firm is only a theoretical link, a mental construct helping to explain how one gets from the cause to the effect. This is altogether different from explaining the behavior of a firm.

4. Discuss the marginalist controversy and its impact on neoclassical price theory.

5. How was neoclassical price theory generalized so as to be able to absorb full cost pricing?

6. How was full cost pricing rationalized so as to be consistent with marginalism?

7. The full cost prices described by Hall and Hitch in "Price Theory and Business Behavior" are not set by equating marginal cost to marginal revenue. What implications does this have for neoclassical price theory?

8. The notion of an inflexible, stable, or fixed price has bedeviled economists for the last 70 years because such a price seemed inconsistent with their theory of prices. In light of

this, answer the following questions:

a. What was G. C. Means's explanation for stable short-term prices?
b. Why is neither Sweezy's and Hall and Hitch's kinked demand curve explanation of stable prices not consistent with neoclassical price theory?
c. Assuming that you are a dedicated and high-powered neoclassical price theorist, construct an explanation for stable prices that is consistent with your price theory.

PROBLEM SET 6

Factor Input Markets and Distribution

Marshall's Theory of Distribution

1. What is Marshall's explanation for the determination of wages and interest?

2. Why is Marshall so concerned with the question of distribution?

3. To what degree did Marshall anticipate the marginal productivity theory of distribution?

4. What are some of the problems with Marshall's theory of distribution?

Demand and Supply of Factor Inputs

1. "If labor costs rise by 5% and capital costs rise by 10%, then the firm will hire more workers and reduce its usage of capital." Comment upon whether this statement is either true, false, or uncertain.

2. Suppose that Mr. Carreras is the only decision maker in the firm. The profits of the firm depend solely upon the time he spends on the job: @ = f(H) where @ is profits and H is the number of hours Mr. Carreras spends at work. If Mr. Carreras spends no time at the firm, profits will be zero. Hence for the firm to be profitable, he must spend some time at work. However, if he spends too much time at work, he gets on the nerves of the workers and profits fall. Consequently, the maximum level of profits will be achieved at some level of time that is less than 24 hours. Suppose that Mr. Carreras also has a utility function for profits and leisure (L = 24 - H) of the form U = U(@, L). In this situation, will the number of hours that Mr. Carreras would spend at work to maximize profits be the same number of hours that he would spend to maximize utility? Graphically

demonstrate your answer and describe the conditions under which they are either equal or different.

3. Consider a profit-maximizing firm that employs a single variable input. Suppose that the output, y, for each level of x is given by the following schedule:

y	1	2	7	10	12	13	13	12	10
x	0	1	2	3	4	5	6	7	8

a. Compute the schedules for the average and marginal product.
b. Let $p_y = 2$ and $p_x = 4$, determine the level of output that would maximize the firm's profits. How much of the input is purchased by the firm at this level of output?

4. The production of widgets requires only two inputs. Each morning, potential workers form a line outside the factory. The owner must decide how many laborers he/she will hire for the day. Given the following schedules, determine the profit-maximizing level of the variable input:

variable input	0	1	2	3	4	5	6
VMP	0	100	80	60	40	20	0

price of the variable input = $40.00

5. A small toy-manufacturing plants uses three different assembly processes to produce wooden boats. The marginal productivity of workers in each process is as follows:

marginal product of the ith worker	1	2	3	4	5
process A	20	16	12	8	4
process B	18	16	14	12	10
process C	12	11	10	9	8

The boats can be sold for $10 each.
Workers are paid $120 per day.

a. How many workers should be hired?
b. How many workers should be assigned to each production process?

6. VMP will equal the marginal product multiplied by the price of the good produced. Explain why.

7. A total product schedule is given for a firm that utilizes only one variable input and sells its product in a perfectly competitive market. The price of the good is $5.00. The price of the variable input is $60.00. Find the profit-maximizing quantity of the variable input for the firm.

variable input	0	10	20	30	40	50

total output 0 200 350 470 570 650

8. Demonstrate that the VMP curve will be the demand curve for the variable input when the firm uses only one variable input.

9. To obtain a market demand curve for a good, we summed horizontally the demand curves of all consumers in the market. Explain why we cannot obtain a market demand curve for an input in exactly the same fashion.

10. Why is the demand for factor inputs called a derived demand?

11. Can you relate the price of a product and the demand for a factor input that is used in the production of the product? Discuss.

12. Explain why the MRP curve must be negatively sloped for a firm using a single variable input. Why will a MRP curve be more steeply sloped than a VMP curve for a firm with the same production function?

13. Explain why the MRP curve is the demand curve for labor by a firm when the firm has only a single variable input.

14. Given the following information, answer the following questions:

variable input L	30	40	50	60	70	80
total output		530	730	910	1070	1210 1330
total revenue	5830	7300	8190	8560	8470	7980

a. Compute the MRP schedule; and
b. Find the optimal integer for L for wage rates of $147, $107, and $37.

15. Which of the input combinations listed here will satisfy the conditions for profit maximization? Assume that the price of labor is $1, the price of capital (K) is $2, and marginal revenue is $5:

input combination 1 (8L, 6K) MPL = 5 & MPK = 1
input combination 2 (3L, 2K) MPL = 2 & MPK = .5
input combination 3 (4L, 2K) MPL = .2 & MPK = .4
input combination 4 (2L, 4K) MPL = 2 & MPK = .1

16. Given the following information, answer the following questions:

quantity of labor	10	20	30	40	50	60
wage rate	3	4	5	6	7	8
MRP	19	17	15	13	11	9

a. Calculate the marginal factor cost schedule for labor; and

b. Find the profit-maximizing quantity of labor and the wage rate for a monopsonist.

17. The marginal revenue curve for a monopoly lies below the market demand curve for the good. Explain by analogy why the marginal factor cost curve for a monopsonist lies above the supply curve for the input.

18. A union for skilled workers estimates that the demand curve for its members has the form W = 200 - .01L. Find the wage rate (W) the union should seek if it wishes to ensure that 5000 of its members are employed. Find the wage rate the union should seek if it wishes to maximize the total wage income for its 15,000 members.

19. Using the graphical method, show how an individual firm's demand curve for a factor input under conditions of imperfect competition will differ from that of a firm under perfect competition. Does this suggest anything to you with respect to the relative levels of employment that would be obtained under perfect and imperfect competition? Explain why it is difficult to develop a definitive answer to this question.

Distribution of Income

1. What are the deficiencies of marginal productivity theory as a complete theory of income distribution? How does a consideration of factor supply contribute to the explanation of prices of productive services?

2. Human capital theory has been constructed during the past quarter century in order to advance the neoclassical theory of personal income distribution. What theoretical problems was human capital theory meant to address? How successful has it been in overcoming the theoretical limitations of earlier neoclassical theories of personal income distribution?

PROBLEM SET 7

General Equilibrium and Welfare Economics

General Equilibrium

Production and Resource Allocation

1. Define scarcity and spell out the implications of this concept.

2. Land-Labor production model

   a. What are the parameters of the model?
   b. What are the constraints of the model?
   c. What are the variables to be determined?

3. Define opportunity cost.

4. Let $A = \begin{bmatrix} 6 & 2 \\ 2 & 1 \end{bmatrix}$, L = 250, and T = 100:

   a. Find Yr and Yw that fully utilize land and labor.
   b. What is the condition for the full employment of land and labor?
   c. What does the full employment of land and labor mean for opportunity costs?

5. Let $A = \begin{bmatrix} 8 & 2 \\ 1 & 2 \end{bmatrix}$, L = 200, and T = 100:

   a. Find Yr and Yw that fully utilize land and labor.
   b. How much land is allocated to the production of rice?
   c. How much land is allocated to the production of wheat?
   d. How much labor is allocated to the production of rice?
   e. How much labor is allocated to the production of wheat?

6. Let $A = \begin{bmatrix} 12 & 4 \\ 5 & 3 \end{bmatrix}$, L = 200, and T = 100:

   a. Find Yr and Yw that fully utilize land and labor.
   b. What is the condition for the full employment of land and labor?
   c. How much land is allocated to the production of rice?
   d. How much land is allocated to the production of wheat?
   e. How much labor is allocated to the production of rice?
   f. How much labor is allocated to the production of wheat?

Price Relations

1. Given the following price model

   $6p_L + 2p_T = p$
   $2p_L + 1p_T = 1$

   and L = 250 and T = 100:

   a. Derive the supply curve for wheat.
   b. Derive the supply curve for rice.
   c. Find p_L and p_T when
      1) p = 1
      2) p = 2
      3) p = 2.5
      4) p = 3
      5) p = 4

2. Given the following price model

   $8p_L + 1p_T = p$
   $2p_L + 2p_T = 1$

   and L = 200 and T = 100:

   a. Derive the supply curve for wheat.
   b. Derive the supply curve for rice.
   c. Find p_L and p_T when
      1) p = .5
      2) p = 1
      3) p = 2
      4) p = 3
      5) p = 4

3. Given the following price model

$12p_L + 5p_T = p$
$4p_L + 3p_T = 1$

L = 200, T = 100:

a. Derive the supply curve for wheat.
b. Derive the supply curve for rice.
c. Find p_L and p_T when
   1) p = .5
   2) p = 1
   3) p = 2

4. What is Walras's Law and what are its implications?

5. Explain why factor prices are scarcity indexes of the factor inputs.

Equilibrium of Supply and Demand

1. General Equilibrium of Supply and Demand

a. Data

production model	price model
$6Yr + 2\,Yw = L$	$6p_L + 1p_T = p$
$1Yr + 2\,Yw = T$	$2p_L + 2p_T = 1$

market demand curve for wheat: $Xw = [(10)(I)(p)]/1 + p$

market demand curve for rice: $Xr = [(10)(I)]/p + p^2$

income of the ith consumer: $I = 10p_L + 5p_T$

# of consumers = 10; L = 100 and T = 50

b. Questions

(1) Find Yr and Yw that fully utilize land and labor.
(2) Find (i) p_T, (ii) p_L, (iii) p
(3) What is the distribution of wages and rent in the economy?

2. General Equilibrium of Supply and Demand

a. Data

production model price model

$12Yr + 4Yw = L$ $12p_L + 5p_T = p$
$5Yr + 3Yw = T$ $4p_L + 3p_T = 1$

utility function for the ith consumer: $U = 4(Yr)^{.5} + 2(Yw)^{.5}$

income of the ith consumer: $I = 2p_L + 1p_T$

# of consumers = 100

L = 200 and T = 100

b. Questions

(1) Find Yr and Yw that fully utilize land and labor.
(2) Derive the ith consumer's demand curves for rice and wheat.
(3) Derive the market demand curves for rice and wheat.
(4) Find (i) p_T, (ii) p_L, (iii) p
(5) Show that when the markets for rice and wheat are in equilibrium, all that is supplied is demanded and that land and labor are fully utilized.
(6) What is the distribution of wages and rent in the economy?

3. Comment on the following statement:

The unique view implicit in neoclassical general equilibrium theory is that, by taking resources and technology given, individual consumption choices can be regarded as determining all the important variables--factor allocations, prices, incomes, and the supply of goods.

4. In what respects is general equilibrium analysis superior to partial equilibrium analysis? Why, then, is partial equilibrium analyses so common within microeconomics?

5. What is Richardson's critique of competitive equilibrium?

6. General Equilibrium of Supply and Demand

a. <u>Data</u>

Production Model Price Model

$6Yr + 2Yw = L$ $6p_L + 2p_T = p$
$2Yr + 4Yw = T$ $2p_L + 4p_T = 1$
market demand curve for rice: $Yr = 100 - 35p$

market demand curve for wheat: $Yw = 140 - (260/p)$

income of the ith consumer: $I = 20p_L + 10p_T$

# of consumers = 10

L = 200 and T = 100

b. Questions

(1) Solve for Yr and Yw.
(2) Show that the opportunity cost of a unit of wheat per unit of rice increases as more rise is produced.
(3) Derive the supply curve for wheat.
(4) Solve for p, p_T, p_L, and I.
(5) If p equals the absolute value of the labor constraint, what is the value of p_T and why?
(6) If p equals the absolute value of the land constraint, what is the value of p_L and why?

7. To what degree does general equilibrium fulfill the following definition of economics:

"Economics is the science which studies human behavior as a relationship between ends and scarce means which have alternative uses."

Welfare Economics

1. Assess the theoretical and political importance of the several causes of market failure.

2. Explain the neoclassical treatment of distributional equity and economic efficiency.

3. To what extent is `efficient allocation of resources' conditional upon how income is distributed. Explain the relationship between these two central functions of an economic system.

4. If a good exerts an externality on another commodity, then it is not the market value of the second good that is affected but the allocation of resources that is upset. Critique.

5. The concept of `profit' plays an important theoretical role in the microeconomic drama about how a private enterprise economy functions. Critically assess the conception of profit that one typically finds in microeconomic theory and the role it plays in constructing that theory.

6. What is a social welfare function? Is it identical or similar to an individual utility function? What are the implications of interpersonal comparisons of utility with respect to a social welfare function?

7. Is the Pareto optimality criterion a social welfare function? Defend your answer.

8. Why is a social welfare function so difficult to construct?

9. Does Pareto optimal mean `the greatest good for the greatest number of people'? Explain.

10. What are the difficulties with the Pareto optimality approach in justifying social change, in particular a redistribution of income from the rich to the poor? Does this imperfection of the approach mean the Pareto optimality criterion is useless? Explain.

11. Under what circumstances is one state of the economy considered Pareto superior to another? Explain.

12. Why is the concept of Pareto optimality so important in welfare economics? What difficulty is involved in relying on the concept of Pareto optimality in welfare economics?

13. Why is perfect competition considered highly desirable according to the Pareto optimality approach in welfare economics? Explain.

Criticisms