PART VII

General Equilibrium and Welfare Economics

CHAPTER 32

HISTORICAL INTRODUCTION: METHODOLOGY AND EQUILIBRIUM

Modeling the economy—historical digression—short.

Empirical content of the model

Role of mathematics in the model—functions which have measurement and new maths in which measurement is not required

Nature of equilibrium—old vs. new

Establishing the model of general equilibrium.

Parameters: input/production/supply side

1. the available quantities of the various resources yielding factor services in production or available for exchange

2. the technology of production specifying the ways in which factor services are transformable into final outputs.

Since factor services are measured as flows within a given time period, while resources are measured as stocks, it must be assumed either that the flows are the given data or, if only stocks are given, that the technical link between stocks and flows is fully specified.

Parameters: demand side

Variables: quantities

Variables: prices

CHAPTER 33

GENERAL EQUILIBRIUM: PURE EXCHANGE MODEL

Leon Walras's Theory of Exchange

To establish Walras's general equilibrium model of exchange, it is necessary to present his conception or view of economics. To arrive at his conception of 'scientific' economics, Walras first divided the universe into two categories: natural phenomena that include all facts and events that result from the play of the blind and ineluctable forces and human phenomena which includes all facts that result from the exercise of human free will and self-awareness. Walras then stated that 'real science' only dealt with natural phenomena while human phenomena fell under the domain of the moral sciences. Since economics would seem to fall under moral science and not real science, Walras was preciously close to stating that economics could not be a real science. However he was able to escape the conclusion in the following manner. First he divided human phenomena into two categories: industry and institutions. In the former Walras included all phenomena that are the manifestations of the human will with respect to natural forces (or things), while the latter included all phenomena that are between people. Next he defined social wealth as all things, material and immaterial that are *scarce*, that is to say, on the one hand, useful to us and, on the other hand, only available to us in limited quantity. By useful, Walras meant any thing (natural phenomena) that is capable of satisfying a want; and by limited quantity he meant things are available to individuals in such quantities that wants or desires are not completely satisfied.

Thirdly, Walras spells out the implication of the notion of scarcity. That is, scarce things are useful things limited in quantity and are appropriable (thus providing a basis for private property). In the first place, these things are amenable to seizure and control, in view of the fact that it is physically possible for a certain number of individuals to gather the entire existing quantity of such a thing for themselves, with none of it left in the public domain. In the second place, those who do this reap a double advantage: not only do they assure themselves of a supply which can be reserved for their own use and satisfaction; but if they are unwilling or unable to consume all of their original supply themselves, they are also in a position to exchange the unwanted for other scarce things which they do care to consume. Secondly, useful things limited in quantity are valuable and *exchangeable*. This property of exchangeability arises immediately from the fact that since all scarce things have been appropriated but their distribution is inappropriate, then there is a need for exchange. Thirdly, scarce things can be produced and reproduced by industry. Finally, Walras argued, since the exchangeability of any good is based on the *natural* condition of scarcity, that is on the natural conditions of being useful and limited in quantity, then any value in exchange, once established, partakes of the character of a natural phenomenon, natural in its origins, natural in its manifestations, and natural in essence. So by limiting theoretical (as opposed to applied) economics to the realm of exchange and value in exchange, that is to the realm of social wealth by itself, Walras concluded that economics was a science.

The implication of Walras's conception of economic as a science is as follows. Walras's definition of scarcity excludes all other kinds of relationships between wants and things that might fall under the domain of economics. A good example of what is excluded is the classical approach to wants and things in which scarcity is not a fact. More to the point is that this notion inhibits the ability of economic theory to deal with growth. To assume that humans want things that are scarce is in contradiction to the previously made assumption that human have freewill. However, if Walras did not assume this, then he could not logically account for exchange. That is, as long as individuals want scarce things, given their endowment of things, they will trade what they have to get more of what they want or do not have. As a result exchange takes on a natural quality (as opposed to being social in nature). Wants are treated as given in a sense part of a person's natural make-up. This natural conception of wants is necessary if scarcity is to be a natural characteristic of things and if exchange is to be natural. Moreover, as we shall see, given wants ensures that the notions of maximization, efficiency, and rationality have meaning.

Walras started his analysis of exchange with the following givens: (1) wants are given (that is determined exogenously to exchange), (2) the objects of exchange, that is the scarce goods or things, are given in the form of the individual's endowments; and (3) individuals exchange goods to the point their satisfaction is maximized. This assumption it should be noted also violated the assumption of free will.

In the first part of <u>Elements</u>, Walras argued that exchange value is derived from scarcity; in part II of <u>Elements</u>, he wants to reverse the argument by showing that exchangeable commodities are scarce. That is, Walras wants to work from observables, that is prices, quantities offered and demanded, demand schedules, etc., in order to show their scarcity foundation. To show this argument most clearly, he uses a two-good exchange model.

To establish his argument, Walras makes the following assumptions:[1] (1) traders hold either a stock of good A or good B; (2) traders holding say B will come to the market to exchange part of it for A if it will maximizes his/her total utility; (3) traders are utility maximizers and each has a separable and additive Strongly separable) utility function of the form

$U_b = \mu_a(y_a) + \mu_b(y_b^T - y_b^o)$ that is trader holds good B and demands good A

where $\mu_a(y_a)$ represents the total utility of consuming y_a amount of good A;

$\mu_b(y_b^T - y_b^o)$ represents the total utility from holding and consuming $y_b^T - y_b^o$ amount of good B,

y_b^T is the original stock of good B, and

y_b^o is the amount offered of good B;

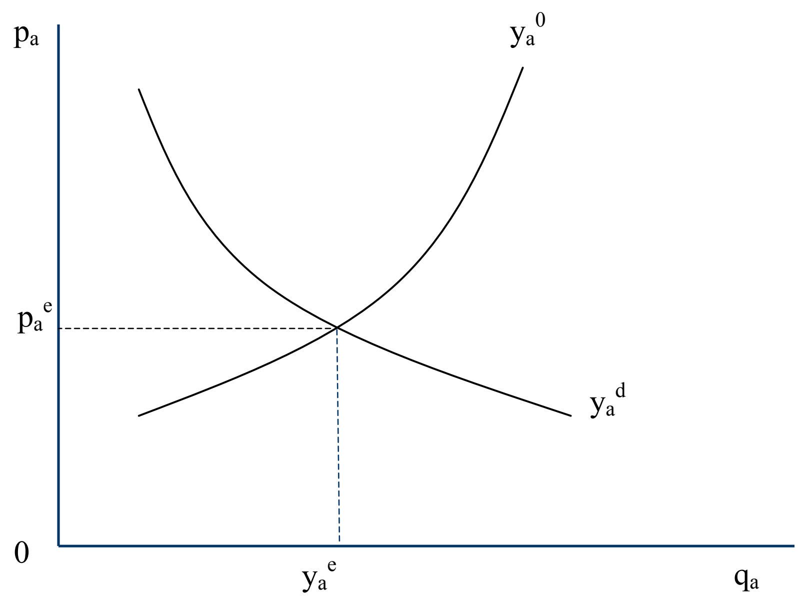
and (4) perfect information exists with respect to the traders' offer and demand schedules.

To start his argument, Walras shows that in equilibrium the amount of offered equals the amount demanded. In a two-good world, if a trader is to demand good A, he must offer good B. Hence in equilibrium $y_b^o v_b = y_a^d v_a$ where v_b and v_a are the "value" of goods B and A and y_a^d is the amount of good A demanded. Rearranging we have $y_b^o = y_a^d p_a$ where $p_a = v_a/v_b$ (also note that $p_b = v_b/v_a = 1/p_a$). Next Walras assumes that the trader's demand schedule is a function of price: $y_a^d = f_a^d(p_a)$ if the trader demands good A. Moreover, Walras referring to empirical evidence (Cournot) assumes that $\partial f_a^d/\partial p_a < 0$. Since $y_a^o = f_b^d(p_b)p_b$, it can be rewritten as $y_a^o = f_b^d(1/p_a)(1/p_a)$ where this represents the offer curve of good A in terms of its own price instead of price p_b. So obviously in

[1] Also see the section 'Equilibrium Firm and the Theory of Prices' in Part VI, chapter 2.

equilibrium when the quantity offered equals quantity demanded, we have $y_a^o = y_a^d$ or $y_b^d(1/p_a)(1/p_a) = f_a^d(p_a)$. This can be shown graphically:

Figure 1



The next step in Walras's argument is to show that the price at which the trader's offer and demand schedules intersect is the price that would maximize his/her total utility. Working with $U_b = \mu_a(y_a) + \mu_b(y_b^T - y_b^o)$, he defines $\partial\mu_a/\partial y_a$ and $\partial\mu_b(y_b^T - y_b^o)/\partial(y_b^T - y_b^o)$ as marginal utility or *rarete'* which he defined as the intensity of the last want satisfied by any given quantity consumed of a commodity.[2] Walras then simply assumes (in contradiction to the notion of free human will) that marginal utility diminishes since the most urgent wants are satisfied first and the less urgent wants later: $\partial^2\mu_a/\partial y_a^2 < 0$ and $\partial^2\mu_b/\partial(y_b^T - y_b^o)^2 < 0$. Next Walras defines the trader's budget as $y_b^T = y_a^d p_a + (y_b^T - y_b^o)$. To maximize total utility, a Langrangian equation is set up:

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[2] *Rarete'* is completely subjective and personal.

$L = U_b = \mu_a(y_a) + \mu_b(y_b^T - y_b^o) + \lambda[y_b^T - y_a^d p_a - (y_b^T - y_b^o)]$.

First order conditions are the following:

$L_{ya} = \partial\mu_a/\partial y_a \partial\mu_a/\partial y_a - \lambda p_a = 0$

$L_{yb} = \partial\mu_b(y_b^T - y_b^o)/\partial(y_b^T - y_b^o) - \lambda = 0$

$L_\lambda = y_b^T - y_a^d p_a - (y_b^T - y_b^o) = 0$.

In equilibrium we have $[\partial\mu_a/\partial y_a \partial\mu_a/\partial y_a - \lambda p_a]/p_a = \partial\mu_b(y_b^T - y_b^o)/\partial(y_b^T - y_b^o) = \lambda$ or $[\partial\mu_a/\partial y_a \partial\mu_a/\partial y_a - \lambda p_a]/[\partial\mu_b(y_b^T - y_b^o)/\partial(y_b^T - y_b^o)] = p_a = \lambda$ or the trader maximizes his/her total utility when the ratio of *rarete* equals the price. To show that p_a is the same equilibrium price that occurs in the equating of the offer and demand schedules, the first order conditions can be solved:

$y_a^d = f_a^d(p_a)$

$y_b^o = f_a^d(p_a)p_a$

(implicitly y_b^T should be included but Walras did not do so). From these two equations, it can be shown that p_a is the equilibrium price that maximizes total utility.

Now Walras is in the position to argue that exchangeable commodities are scarce and that their exchange ratios of two (or more) commodities are based on their relative scarcities or *raretes*. Walras first states that *raretes* (marginal utility) is the same thing as scarcity in terms of the twin conditions of utility and limitation in quantity. There could not possibly be a last want satisfied if there was no "want" for the good—that is, if the good is useless. Moreover, the *rarete* of a good can only be positive if the good is limited in quantity. Therefore exchangeable goods are scarce. Moreover, Walras argued that, given wants-goods matching, variations in the given quantities of a particular good will directly affect its price—increase in quantity decreases its limitness thus decreasing its *rarete* to the trader hence decreasing its price. Thus Walras can argue that exchange ratios, that is prices, reflect the relative scarcity of exchangeable goods and that scarcity is the foundation upon which the theory of exchange is based. This last point is important in that Walras has shown that the assumed desired to maximize satisfaction via scarce thing is the basis of exchange.

Walras's theory of exchange in the 2-good 2-individual case was a theory of exchange in isolation. However, he went on to argue that the results he obtained in this case were precisely duplicated if the situation was altered so that there existed **n** markets each with **m** traders. Walras set up his system of equations for the expanded case and arrived at a mathematical solution for prices and quantities. Then he asked the question as to whether and under what conditions these mathematical solutions could be realized by the market. His answer to this was phrased in terms of an empirical possibility or feasible desideratum rather than an empirical fact and it was of course his famous *tatonnement* process. In developing his argument, he discovered that he had to eliminate the problem of false trades, that is trading at false prices or non-equilibrium prices. The consequence of trading at false prices is that the resulting redistribution of endowments will be such as to affect what the final equilibrium position will be. That is, if trading at false prices is regarded as essential to real *tatonnement* in actual markets, whatever equilibrium is arrived at in the competitive market via *tatonnement* cannot be the same as the equilibrium determined mathematically from a system of equations. In the mathematical solution, the parameters to be held constant are not only tastes, total endowments, but also the distribution of wealth among the traders, that is the values of the sum of the initial quantities possessed by each of the traders. When this last parameters is allowed to shift, as it must when trading at false prices takes place, the equilibrium (if it exists) is unlikely to remain unchanged.

As a result of this problem, Walras developed a timeless, simultaneous and mechanical *tatonnement* process. In the first stage of the process (model), Walras assembled in advance a complete array of partial equilibrium solutions for each market, so ordered that, whatever its starting point, each successive solution in the array cumulatively incorporated the results of the preceding solutions. Secondly, as he could not integrate these separate parts into a comprehensive unified overall model without allowing for mutual feedback effects, Walras assumed that the indirect effects cancelled each other out so that on the whole the system moved towards equilibrium (this assumption is in fact inconsistent with his assumptions, especially his utility functions and is also just wrong).

Walras finishes his discussion of exchange by criticizing Adam Smith's proposition that labor is the origin of value:

> Surely, if labor has value and is exchangeable, it is because it is both useful and limited in quantity, that is to say because it is scarce. Value, thus, comes from scarcity. Things other than labor, provided they are scarce, have value and are exchangeable just like labor itself. So the theory which traces the origin of value to labor is a theory devoid of meaning rather than too narrow…. [Walras, Elements, 1954, pp. 201 – 202]

Walras could say this because he was pointing out a particular feature of classical political economy in which the drive to accumulate involves the allocation of quantities of labor to various sectors of the economy. However Walras misunderstood this allocative mechanism in that in classical political economy labor has value because of the capitalist need to allocate it for growth—the notion of scarcity has nothing to do with it. Thus exchange values in classical political economy comes from the need to accumulate as fast as possible through creating a surplus and since all relative prices are reduced to labor of various sorts in classical political economy, they are based on the notion that labor has exchange value. (It should be noted that Walras's notion of scarcity-based value is completely opposite/different in a different universe then value in classical political economy).

Modern Theory of Exchange

CHAPTER 34

GENERAL EQUILIBRIUM: EXCHANGE AND PRODUCTION WITH PRODUCED INPUTS

Introducing Produced Inputs into the Theory of Exchange

In neoclassical theory the need to introduce the notion of capital into their analysis is not at first all that obvious. That is, in the simple neoclassical exchange model all the agents just need to exchange their endowments (minus what they keep) to get goods they want to consume immediately. Thus why would any of these agents temporarily postpone their consumption via production using capital goods? The answer on quite a simple level is, of course, that production with capital goods enables the agents to turn out an increased amount of consumable goods or even different kinds of consumable goods. Thus we must agree with Bohm-Bawerk when he states:

> The task of formulating the theory of capital as a tool of production consists in describing and explaining capital's entrance upon the stage of economic production and depicting the effects of that entrance.

This role of capital as an intermediary between initial endowments-consumption and future consumption gives it a characteristic of time; on the other hand, since capital is used to increase output, it also has a property of being productive. Bringing these two properties together 'capital' as an abstract homogeneous concept can be envisioned as a 'fund of money' which earns (produces) interest—thus it could in this sense be seen as an original factor of production. However this discussion is bringing us far beyond where we need to start from if we are going to understand the role of capital in general equilibrium models that deal with production.

The purpose of this chapter is not to formulate a theory of capital *per se*; rather it is to show how capital was used in general equilibrium models and tentatively explore some of the essential properties of capital. Specifically, we shall first consider the Bohm-Bawerk, Wicksell and the Austrian approach to capital and its role in production. Then we discuss John Bates Clark analysis of capital dealing with his ‘Clarkian’ corn model. We then deal with Walras’s multi-good general equilibrium model; and the chapter ends with a discussion of the problems of capital goods in general equilibrium models. The essential properties of capital that will be explored are the following: (1) capital as a quantity that can be measured independently of the price of capital (which is the rate of interest), (2) the inverse relationship between the capital-labor ratio (K/L) and the rate of interest, (3) the inverse relationship between the capital-output ratio (K/Q) and the rate of interest, and (4) whether value of the marginal product of capital equals its price the rate of interest.

Production and Produced Inputs: Bohm-Bawerk, Wicksell, and the Austrian Approach

Throughout the history of capital theory, much controversy has been generated regarding the ‘essence’ of capital. The Austrians—William Stanley Jevons, Bohm-Bawerk, and Knut Wicksell—conceived of this essence as resulting from capital goods being produced means of production distinguishable from land and labor which were classified as ‘original’ means of production. Coupled with the distinction was the Austrian emphasis on capitalistic production requiring time. Capitalistic production requires that the production of capital goods precede the production of consumption goods. It was the hallmark of Austrian capital theory to link these two characteristics together. Bohm-Bawerk stated the Austrian position as follows:

> We put forth our labor in all kinds of wise combinations with natural processes. Thus all that we get in production is the result of two, and only two, elementary productive powers – Nature and labor. There is no place for a third primary resource – such as capital. But through these primary productive powers man may make the consumption goods he desires, either immediately, or through the medium of intermediate products called capital. The latter method demands a sacrifice of time, but it has the advantage in the quantity of product, and this advantage, although perhaps in decreasing ratio is associated with every prolongation of the roundabout way of production. [NEED TO GET REFERENCE]

Thus, in the Austrian view there are two types of original productive power, land and labor. In fact, Bohm-Bawerk simplified his analysis by abstracting from land and regarding labor as homogeneous. Capital goods are goods produced with the aid of original factors and are used as intermediate inputs in the production of consumer goods. Capitalistic production is, therefore, indirect or 'roundabout' production. It is undertaken because it is more productive of consumption goods than is direct production.

The Austrians worked with two kinds of production and price models: a point input – point output model and a flow input – point output model. The former is a one production period model in which labor directly creates, without the intervention of capital goods, a consumption good, while the latter is a flow of inputs at various dates but with output emerging at a single date. The production and price models of each are given in Table 1. This view of production is called a linear view because the original input of labor can be traced unilaterally to its final resting place in the consumption good.

Table 1

Point Input – Point Output Model

Production Model

$L_c \rightarrow C$

where L_c is the number of homogeneous labor hours needed to produce C amount of the consumption good.

Price Model (i)[3]	*Price Model (ii)*
$L_c w = Cp_c$ or $l_c w = p_c$	$L_c w(1 + i) = Cp_c$ or $l_c w(1 + i) = p_c$

where w is the wage rate,
p_c is the price of the consumption good,
l_c is L_c/C the input-output or the labor production coefficient that represents the amount of labor needed to produce a unit of the consumption good, and
i is the rate of interest (or rate of profit).

Flow Input – Point Output Model

Production Model

$L_m \rightarrow M_c$
$M_c + L_c \rightarrow C$

where L_m is the number of homogeneous labor hours needed to produce M_c number of machines that are needed to produce C amount of the consumption good, and
L_c is the number of homogeneous labor hours needed to work with M_c number of machines to produce C amount of the consumption good.

Price Model (i)	*Price Model (ii)*
$L_m w = M_c pm$	$L_m w(1 + i) = M_c p_m$
$M_c p_m(1 + i) + L_c w = Cp_c$	$[M_c p_m + L_c w][1 + i] = Cp_c$
or	
$l_m w = p_m$	$l_m w(1 + i) = p_m$
$m_c p_m(1 + i) + l_c w = p_c$	$[m_c p_m + l_c w][1 + i] = p_c$

[3] The difference between the two price models is whether wages are advanced or not; that is whether wages are part of the capital advanced or come out of the surplus. MORE ON THIS.

where w is the wage rate,
p_m is the price of the machine,
p_c is the price of the consumption good,
l_m is L_m/M_c the input-output or labor production coefficient that represents the amount of homogeneous labor hours need to produce M_c number of machines that are needed to produce C amount of the consumption good,
m_c is M_c/C the input-output or the machine production coefficient that represents the number of machines needed to produce a unit of the consumption good,
l_c is L_c/C the input-output or the labor production coefficient that represents the amount of labor needed to produce a unit of the consumption good, and
i is the rate of interest (or rate of profit).

Austrian Model of Production With Capital

The purpose of the Austrian model of production was to develop a theory of interest that was opposed to the Marxian-Classical theory of profits. Working from Karl Menger's utility theory and integrating it with his vertical ordering of goods and Jevons's conception of capital as advances to workers for production, Bohm-Bawerk developed the following theory of capital and interest. He developed a theory of interest at two different levels of abstraction. At the most abstract level, we have a psychological theory of interest. Economic agents are assumed to have a time preference for present consumption relative to consumption in the future. Consequently, inter-temporal exchange ensures that a premium accrues to those who trade present for future consumption. Two principle conclusions derived from this analysis are interest arises from exchange and that it is a universal economic category that is not historically specific to capitalism.[4] Bohm-Bawerk presented the following argument for positive time preference and hence the existence of interest (profit):

> The great frequency of desire for present goods to meet the current needs of people who will be more adequately provisioned with goods in the future.

[4] Hence Marx's theory of profits is thereby questioned at its foundation.

Therefore we systematically undervalue our future wants and also the means which serve to satisfy them because of the fragmentary nature of the imaginary picture that we construct of the future state of our wants, the failure of willpower, and the consideration of the brevity and uncertainty of human life. Therefore the greater productivity of more roundabout methods of production leads to a physical and value surplus over what was used up in production. [WORK ON]

At the second level of abstraction, Bohm-Bawerk applied his theory of interest to the institutions of capitalism. Here it is the strength of labor's time preference for present consumption relative to that of capitalists' which ensures interest for the latter. Capitalists can advance consumption goods to workers in the form of wages, engage them in roundabout production processes, and thereby receive a premium on advances made. To show this, let us consider the following argument.

The capitalists have a supply of a consumption good—say corn—of the amount K to start the production cycle. The total amount of labor to start the production process is L and the workers need to buy corn in order to survive the production process. Thus $Kp_c/L = w$ or $Lw = Kp_c$ or $(L/K)w = p_c$ where p_c is the price of corn and w is the wage rate. Working with a two-stage flow input-point output production/price model

$$L_m w(1 + i) = M_c p_m$$
$$[M_c p_m + L_c w][1 + i] = Cp_c$$

Now reducing the price model to dated quantities labor via substitution and adopting Bohm-Bawerk assumption that interest accrues on the basis of simple interest (as opposed to compound interest) [NEED REFERENCE], we have

$$L_m w(1 + 2i) + L_c w(1 + i) = Cp_c$$

Since labor is assumed to be homogeneous the dated labor price model can be rewritten as:

$wL + w(2L_m + L_c)i = wL + wKi = Cp_c$ where $K = 2L_m + L_c$ represents the quantity of capital—more below.

Now assuming $p_c = 1$ and rearranging, then i and w can be written as a wage rate-interest rate relationship: $i = \underline{C - wL}/wK$. Now for a given quantity of labor and corn or L and K and assuming that the roundabout production is productive (that is C > K), it is possible to determine w and i. In this context, the interest exists because not all the corn that is produced is immediately consumed by workers.[5]

[MORE/WORK ON]

Given his theory of interest, Bohm-Bawerk went on to argue that technically efficient production processes are ordered by their degree of rountaboutness. The more roundabout production processes are the more productive of consumption goods per unit of original factor input (which is labor since land has been abstracted from), but are subject to diminishing returns. That is, an increase in roundaboutness relative to the inputs of original factors generates an incremental increase in final output but at a decreasing rate. Pivotal to this conception is the definition of the degree of

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[5] This can be showed in the following example.

Price model: $10Lw(1 + i) = 5_m p_m$

$(5_m p_m + 5Lw)(1 + i) = 30_c p_c$

Adopting Bohm-Bawerk' assumption that interest accrues on the basis of simple interest (as opposed to compound interest) and assuming $p_c = 1$, the price model can be rewritten as

$10Lw(1 + 2i) + 5Lw(1 + i) = 30_c p_c$ or

$15Lw + 10Lw2i + 5Lwi = 30_c$ or

$w(15L + 25Li) = 30_c$ where 25L is the amount of labor that is considered capital and 15L is the amount of direct or 'living' labor used in production. Since $Kp_c/L = 25L/15L = w$, then w = 1.67. With the wage rate know, the interest rate can be determined by the wage-interest rate relationship: $w = 1/(.5_{lc} + .833_{lc}i)$ or i = 12%.

roundaboutness and the determination of the quantity of capital. Bohm-Bawerk defined the degree of roundaboutness (or the length of ‘time’ between the first original input, labor, and the last output) in terms of what he called the *average period of production* (APP):

$$APP = \frac{PP_n L_n + \ldots + PP_1 L_1}{L_n + \ldots + L_1} = \sum_{j=1}^{n} L_j j / \sum_{j=1}^{n} L_j$$

where the numerator represents the sum of the original factor inputs (labor) weighted by the time in which they remain in production;

the denominator is the unweighted sum of these factor inputs; and

APP expresses the average period that inputs are required in the production before the emergence of final input.[6]

Turning to the quantity of capital, the cost of producing the consumption good can be written as $L_n w(1 + ni) + \ldots + L_1 w(1 + i) = Cp_c$. Since the wage bill can be written as $\sum_{j=1}^{n} L_j w$ and total interest (or profit) can be written as $\sum_{j=1}^{n} L_j w(1 + ji) - \sum_{j=1}^{n} L_j w = iw\sum_{j=1}^{n} jL_j$.

Since i is the rate of interest, $w\sum_{j=1}^{n} jL_j$ is the value of capital, $\sum_{j=1}^{n} jL_j$ is the quantity of capital in terms of ‘dated’ labor.[7]

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[6] Working with the price model in the previous footnote the average period of production is:

$$APP = \frac{(2)(10L) + (1)(5L)}{10L + 5L} = \frac{25L}{15L} = 1.67.$$

[7] Working with the price model in footnote 5 we have $10Lw(1 + 2i) + 5Lw(1 + i) = 30cp_c$. Subtracting out the wage bill we have $10Lw(1 + 2i) + 5Lw(1 + i) – 15Lw = i25Lw$ which is total interest; thus 25Lw is the value of capital and 25L is the quantity of capital in terms of dated labor.

Now we are in a position to make the following statements. First, production processes can be ranked in terms of their roundaboutness with respect to their productiveness. Since all techniques can be denoted or marked in terms of their capital-labor ratio, K/L, we can state as did Bohm-Bawerk did that the greater the K/L ratio the greater the output. However, this is the same as stating that the greater the average period of production the greater the output since

APP = $\sum_{j=1}^{n} jL_j / \sum_{j=1}^{n} L_j$ = K/L that is the average period of production is the capital-labor ratio in terms of labor. Thus the APP is a measure of both the degree of roundaboutness and the K/L ratio or the degree of capital intensity. Secondly, an increase in the APP relative to labor (meaning an increase of K relative to a given quantity of L) increases output but at a decreasing rate—that is the marginal product of capital declines [MORE ON THIS]. From these two statements, Bohm-Bawerk made a third significant statement in that the greater the APP (or K/L), the lower the interest rate and that the wage rate and the interest rate are inversely related. To show this, let us go back to our integrated price model:

$$L_n w(1 + ni) + \dots + L_1 w(1 + i) = Cp_c \text{ or}$$

$$\sum_{j=1}^{n} L_j w + iw \sum_{j=1}^{n} jL_j = Cp_c.$$

Now setting $p_c = 1$, substituting in L and K for the aggregate quantity of labor and capital. and rearranging we get the wage rate-interest rate relationship:

$$i = \frac{C}{wK} - \frac{wL}{wK} = C/(wK) - 1/APP = APP^{-1}(C/Lw - 1)\text{—WORK ON.}$$

Now as the average period of production increases, meaning K increases relative to L remaining constant, the interest rate declines; and it also declines because C/K declines due to the declining marginal productivity of capital. And as the interest rate declines the wage rate increases. Thus there is an inverse relationship between the wage rate and the interest rate; and in addition, there is an inverse relationship between the interest and the average period of production or K/L as well as the demand for capital—that is the demand curve for capital is negatively sloped.

<u>Knut Wicksell's Model of Production with Capital</u>

Wicksell developed Bohm-Bawerk's model of production with capital goods, but in doing so he made some changes and ran into a number of problems in discussing prices and distribution. To construct his long period general equilibrium model of production, Wicksell established the following parameters. First the analysis takes place in terms of a stationary state analysis—that is the economy is assumed to engage in periodic renewal and replication. Such a method of analysis is a specific kind of long period analysis. Secondly, the net output consists only of consumption goods (if there was a new output of capital goods, the economy would not be in a stationary state). Finally, labor gets paid at the end of the production period and thus partakes, with 'capital' in the distribution of the net output or surplus. Consequently, there is no 'mark up' on wage costs—see price model (i) above.

Wicksell starts out his analysis by considering, implicitly, a purely capital-labor model of the sort that involves production as a circular process and then inquiries into the relationship between the marginal product of capital and the rate of interest. Wicksell assumes the 'law of marginal productivity' governs the wage rate in that the larger the total amount of labor in the economy is the lower is its marginal productivity and hence its wage rate. Thus there is a downward sloping demand curve for labor. However, with respect to capital as a whole or *social capital* (as opposed to capital associated with a specific entrepreneur), the marginal product of capital does not regulate the rate of interest:

> If we consider an increase in the total capital of society, then it is by no means true that the consequent increase in the total social product would regulate the rate of interest. In the first instance, new capital competes with the old and thereby results in a rise of wages, possibly without causing much change in the technical composition of the product or the magnitude of the return. For this reason, interest must certainly fall, even if the additional product of the new capital is almost nil. The increase in wages may absorb the superfluous capital, so that the latter is now just sufficient for the needs of production, in spite of the fact that production has in reality scarcely expanded at all. [Wicksell, 1977, pp. 148 – 149]

The reason that Wicksell gave for this is that, whereas labor is measured in technical units, capital is reckoned in *value terms* which is extraneous to itself. Continuing, Wicksell argued that the productive contribution of a piece of technical capital (a machine) is determined not by its cost but by the 'service' that it develops, and by the excess or scarcity of similar machines. If capital also were to be measured in technical units, the defect would be remedied and the correspondence would be complete. But, in that case, productive capital would have to be distributed into as many categories as there are kinds of tools, machinery, and materials, and a *unified treatment of the role of capital in production would be impossible*. Even then we should only know the yield of the various objects at a particular moment, but nothing at all about the value of the goods themselves, which it is necessary to know in order to calculate the rate of interest, which in equilibrium is the same on all capital. Again, it is futile to attempt to derive the value of capital goods from their own cost of production or reproduction since the cost of production includes *capital* and *interest* and thus we would be arguing in a circle.

To illustrate Wicksell's above concerns, let us consider the following capital-labor model:

$$(12_m p_m + 120_c p_c)(1 + i) + 10Lw = 20_m p_m$$

$$(\ 8_m p_m + 280_c p_c)(1 + i) + 60Lw = 575_c p_c$$

where m is machine;

c is corn;

i is the rate of interest;

w is the wage rate; and

p_c and p_m are the prices of corn and machines respectively.

The model has two equations but four unknowns: interest rate, wage rate and the prices of corn and machines. Thus it is necessary to exogeneously determine the values of two of the unknowns in order to determine the other two. If p_c is taken as the numeraire and set equal to one, then the price of the machine cannot be determined independently of the wage rate or the rate of interest. Moreover, the total physical quantity of capital used in production—$20_m p_m + 400_c p_c$—cannot be aggregated into a single quantity except in terms of value. Thus the total amount of capital is dependent on the wage rate and the rate of interest; and at different values of the interest rate and wage rate, the value quantity of capital can change even though the physical quantities remain the same. This can be shown from the price model (with $p_c = 1$):

i = 25%	w = \$0.00	total capital = \$700.00
i = 20%	w = \$0.5365	total capital = \$687.24
i = 15%	w = \$1.0535	total capital = \$675.09
i = 10%	w = \$1.5522	total capital = \$663.43
i = 5%	w = \$2.03	total capital = \$652.30
i = 0%	w = \$2.50	total capital = \$641.65

The point to notice is that the fall in the interest rate resulted in a decline in the value quantity of capital although the physical quantities remain the same—this is called a negative price Wicksell effect (there are also positive and neutral price Wicksell effects which shall be discussed later). Such an occurrence counters all intuitive understanding of the relationship between the interest and the quantity of capital. Moreover because capital is in value terms, its 'marginal product' cannot be the regulator of the interest rate since the former already includes the latter. In addition, as can be seen from the negative price Wicksell effect, the 'quantity' of capital can vary directly with the interest rate, thus implying the 'real' increases in the quantity of capital can not regulate it and, moreover is dependent on it (and the wage rate). That is, the quantity of capital cannot be determined independently of the interest rate; and hence there may not be a demand curve for capital—a point that will be discussed later.

Wicksell sought to escape these problems by adopting the Austrian production model in which all capital is reduced to dated quantities of homogeneous labor. Assuming that workers get paid out of the surplus, this means that capital consists only of dated labor embodied in machines and not the direct labor used in production. Given this, he starts his analysis with a two-period price model:

$$L_m w = M_c p_m$$
$$[M_c p_m][1 + i] + L_c w = Cp_c$$

From the model the average period of production can be derived:

$$APP = \frac{(1)L_m + (0)L_c}{L_m + L_c} = \frac{L_m}{L}$$

Now the quantity of capital is defined as $L_m[1 + i] + L_c w - L_m w - L_c w = iL_m w$. So $L_m w$ is the value quantity of capital and L_m is the quantity of capital in terms of dated labor which is determined independent of the interest rate. Therefore the APP measures both the degree of roundaboutness and the capital-labor ratio. Finally, the wage rate-interest rate relationship can be derived from $L_m[1 + i] + L_c w = Cp_c$. Setting $p_c = 1$, we get $i = \frac{C}{wK} - \frac{wL}{wK} = C/(wK) - 1/APP = APP^{-1}(C/Lw - 1)$. Thus, in this two-period price model, Wicksell gets the same results as Bohm-Bawerk. However, these results Wicksell found out cannot be extended to a three-period price model because of the compounding of the interest rate.

When extending his analysis to a three-period price model, Wicksell presented it as though little if any problems existed. However he did not that in doing so the capital-labor ratio became a function of the interest rate and hence could not be represented by the APP or form part of the argument relating the interest rate to different quantities of capital. Moreover, even the quantity of capital in this case becomes a function of the interest rate and hence impossible to measure. Consequently, all the problems Wicksell noted in the pure capital-labor model discussed above re-emerge. To show the problem with the average period of production and the quantity of capital, let us consider the following price model:

$$L_m w = M_g p_m$$
$$[M_g p_m][1 + i] + L_g w = G_c p_g$$
$$[G_c p_g][1 + i] + L_c w = C p_c$$

where G_c is the number of machines needed to produce C amount of consumption goods; and

M_g is the number of machines needed to G number of machines whict are needed to produce C amount of consumption goods.

From this the quantity of capital can be derived as follows:

$L_m w(1 + i)^2 + L_g w(1 + i) + L_c w – L_m w – L_c w – L_g w =$

$i(2L_m w + iL_m w + L_c w)$ = total profits = iK where K is the value quantity of capital; hence K can be denoted as $w(2L_m + iL_m + L_c) = wQ_k$ where Q_k is the 'physical' quantity of capital.

But Q_k is a function of the interest rate and hence cannot be determined independently of it. Now turning to the average period of production, we have:

$$APP = \frac{(2)L_m + (1)L_g + (0)L_c}{L_m + L_g + L_c} = \frac{2L_m + L_g}{L}$$

However since the capital-labor ratio is Q_k/L, it is not the same as the APP by the factor of iL_m which means that the APP cannot stand as a proxy for different amounts of capital. Finally, let us consider the wage rate-rate of interest relationship. Working with $L_m w(1 + i)^2 + L_g w(1 + i) + L_c w = Cp_c$ and letting $p_c = 1$, we get

$w = \frac{C}{L + iQ_k} = \frac{C}{L[1 + i(Q_k/L)]}$. Thus, while the wage rate and the interest rate are inversely related, it is not clear that the capital-labor is inversely related to the interest rate which implies that there may be no montonically declining relationship between the interest rate and the average period of production. Thus, Bohm-Bawerk arguments cannot be extended to a price model in which the interest rate is compounded. MORE ON THIS

Let us consider another extension of Wicksell's model. The model is a two-period price model in which two capital goods are used to produce a consumption good. The price model can be represented in the following manner:

$$L_m w = M_c p_m$$
$$L_g w = G_c p_g$$
$$[M_c p_c + G_c p_g][1 + i] + L_c w = C p_c$$

Now consider the following relationships (assuming $p_c = 1$):

(1) $M_c = g(L_m)$ thus $\partial M_c/\partial L_m = w/p_m$;

(2) $G_c = h(L_g)$ thus $\partial G_c/\partial L_g = w/p_g$;

(3) $C = f(M_c, G_c, L_c)$ thus $\partial C/\partial L_c = w$;

$$\partial C/\partial M_c = p_m(1 + i);$$

$$\partial C/\partial G_c = p_g(1 + i);$$

(4) $L = L_m + L_g + L_c$

(5) $i = [\partial C/\partial M_c - p_m]/p_m = [\partial C/\partial G_c - p_g]/p_g$

Thus we have ten equations—three from the price model and seven from above—and eleven unknows—L_m, L_c, L_g, L, M_c, G_c, C, w, p_m, p_g, and i. Thus to close the model, Wicksell has to add the following equation:

(6) $M_c p_m + G_c p_g = K^*$ = social capital in value form.

The equation implies that the demand for social capital exhausts the economy's endowments of capital. With the addition of this last equation, the solution for the eleven unknowns is now possible. However a solution is not possible because K* depends on the interest rate, p_m, and p_g, three variables we are trying to determine. Thus we are trapped in a circular argument in that we need to know K* to determine p_m, p_g, and i and we need to know p_m, p_g, and i to determine K*. If K* is not in fact given, then it must be dropped thus making it impossible to solve for the unknowns.

Thus, as we see, when Wicksell's model is extended beyond its most simple form the quantity of capital cannot be measured and nor can solutions for the models be found or if found do not maintain the relationship found in the simple model. In particular, the marginal product of social capital cannot be said to regulate the interest rate and moreover the demand curve for capital is problematical. [Even further it can be argued that capital cannot be considered a scarce factor input and therefore does not belong in the general equilibrium—this point will be discussed later on.] We shall now turn to J. B. Clark who developed a model in which the above problems with capital are over come. Also many of these problems will be dealt with again in the next section.

Production and Produced Inputs: Clarkian 'Corn' Model

Clark took a somewhat different approach towards capital than did the Austrians. First of all he distinguish between concrete physical capital-goods and capital as a fund of goods or a sum of money:

> Capital goods imply waiting for fruits of labor. Capital, on the contrary, implies the direct opposite of this: it is the means of avoiding all waiting. It is solidified time, or the material results of waiting on a vast scale. [NEED REFERENCE]

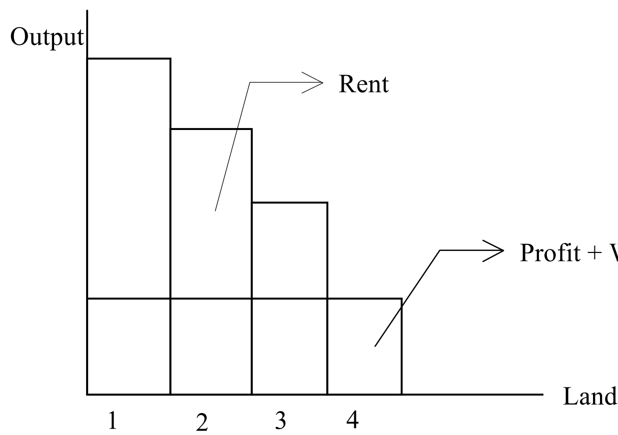
But here Clark deposits markedly from the Austrians for he identifies this solid 'thing' capital as distinct factor of production. The physical aspect of capital thus highlights the fixity of capital at any point in time and suggests to Clark the possibility that its return can be treated just like any other fixed factor—such as land:

> Tools are productive, but time is the condition of getting tools—this is the simple and literal fact. The round-about or time-consuming mode of using labor insures efficient capital-goods. Granting that time be used for this purpose, we may say that 'time is productive'; but we must be careful to keep in view the fact that it is the tools secured by time that do the producing. [GET REFERENCE]

Thus having paid lip-service to the importance of time, he articulates the productivity of long processes not to time but to the goods themselves. Since the tools cannot be anything but fixed at any point of time, Clark sets out to develop a general theoretical system by applying Ricardo's theory of rent to capital and labor to determine the distribution of income as well as the dynamic path of economic development. To carry out such a task, capital had to be considered as a unique quantity that could, as the theory required, be held constant as labor varied, and become a variable when labor was the fixed factor

As with Marshall, Clark adopted the Ricradian approach to rent. As noted in the discussion of classical political economy and with Marshall's theory of production, Ricardian rent can be approached in terms of extensive rent and intensive rent. In extensive rent lands of different quality are cultivated side by side so that, say, in a photographic view, all the lands are lands are observed to be simultaneously in cultivation, the differential productivities of land and the rents arising therefrom can be directly worked out on the basis of the single observed situation. The 'no rent' land is distinctly and concretely identifiable as distinguished from those producing surplus—see Figure 1. On the other hand, in the case of extensive rent, we have the application of

Figure 1



successive doses of capital and labor to the same piece of land—see Figure 2. As a result the ‘intensive margin’ is different than the ‘extensive margin’ above. The marginal product here refers to the incremental return to an additional dose of labor and capital. In order to be identified, the ‘marginal output’ requires a quantitative change in the situation.

Figure 2