(2) interest rate =savings/income

(3) income = 470p+1250 / 24 +9.125p + 31.375

consumption = 9.125 p + 31.375

savings = 470p+1250 / 24

(4) i= (19.583p + 52.0833) / (28.708p + 83.45833)

c. Solving for the unknowns

(1) Using the above p_m / i+d equation where i = (4.35p – 1.35) / (1.3p +.7) and equation (4) above we can find p=.508 and i_m=.632

(2) Using them we can solve for

p^*_m =.0641667; p^*_n = .1871667; i_n = 2.99

d. thus the rates of interest are not uniform.

General Equilibrium, Production, and Produced Inputs: Problems and Attempted Solutions

Joan Robinson Poses a Nasty Problem

Prior to 1953 neoclassical theory had developed a number of parables, dressed up in terms of rigorous mathematical models, to explain the distribution of income. These parables, while developed in a very simplistic manner, were intended to provide a true picture of how the economy operated. Before going on to investigate the theoretical problems involved with these parables, let us summarized them first. Our discussion will be based on the J.B. Clark one commodity capital labor model.

a. The neoclassical parables are set out in terms of a one commodity model in which putty is both the output and the capital good. At the center of the model is a production function which is linearly homogeneous and of the form of P^aL^{1-a}=Y.

The production function is continuously differentiable with positive and diminishing marginal products of P and L:

(i) $ЭY/ЭL= (1-a)\ (P/L)^a$

(ii) $Э^2Y/ЭL^2=(-a)\ (1-a)\ P^aL^{-a-1}<0$

(iii) $ЭY/ЭK=a(P/L)^{a-1}$

(iv) $Э^2Y/ЭK^2=(a)(a-1)P^{a-2}L^{1-a}<0$

(v) if P=0 Y=0, if L=0 Y=0

b. Now given this technology and facing a perfectly competitive markets, the equilibrium position of the economy as a whole will exist when firms choose that technique of production (i.e. a P/L ratio corresponding to a point on the production function) which maximizes profits. This can be shown in the following manner:

(1) Let us assume that the putty is infinitely durable (this assumption does not affect any of the results; it just makes the exposition easier)

(2) Setting up our Langragian, we get $L=P_{pc}i+L_w+\lambda(y^0 - P^aL^{1-a})$

(3) Now remember that $P_{pc}i+L_w=YP_L$ since we are setting $p_c=1$ we get the following

(4) $L=P_i+L_w+ \lambda(y^0 - P^aL^{1-a})$

(5) First order conditions

(a) $L_p=p_{=I} - \lambda a(P/L)^{1-a}=0$

(b) $L_L=w- \lambda(1-a)\ (P/L)^a=0$

(c) $L_{\lambda}= y^0 - P^aL^{1-a}=0$

(6) Now it can be shown else where that λ=marginal costs=p_L in a perfectly competitive system – thus $\lambda=1$. Therefore we have the following results;

(a) $i=MP_p=a(P/L)^{a-1}$

(b) $w=MP_L= (1-a)(P/L)^a$

c. From these results we can now derive the following results

(1) Demand curve for putty (solving first order conditions)

$p= y^0/ [(i/w)^{1-a} (1-a/a)^{1-a}]$ and $\partial p/\partial i<0$, so the demand curve for putty declines.

(2) Demand curve for labor (solving first order conditions)

$L=y^0 (i/w)^a[(1-a)/a]^{1-a}$ and $\partial L/\partial w<0$.

(3) Distribution of income

(a) Euler's theorem: this theorem states that if each factor input is paid the amount of its marginal product, the total product will be exactly exhausted by the distributive shares for all the factor inputs:

(i)$Y=L_w+iP$ now Euler's theorem states that

(ii) $Y\equiv L\ MP_L+P\ MP_K$ where $MP_L=w$ and $MP_K=i$.

(iii) so substituting we get

$$Y\equiv L\ (1-a)P^aL^{1-a}+P(a)P^{a-1}L^{a-1}$$

$$Y=L(P/L)^a\ [(Pa)/L(P/L) +1-a]$$

$$Y=L(P/L)^a[a+1-a]$$

$$Y=L(P/L)^a = P^aL^{1-a}$$

(b) wage-interest rate frontier

Let us work with the following relationship

$$W=(1-a)(P/L)^a=(P^aL^{1-a})/L - a(P/L)^{a-1}\ (P/L)\text{ or}$$

$$W=Y/L -iP/L\text{ since }a(P/L)^{a-1}=MP_p$$

(c) shape of the wage-interest rate frontier

* $-dW/di=P/L<0$ which also states that there is an inverse relationship between P/L and i.

(d) elasticity of the wage-interest rate frontier

$-sdW/Wdi=iP/WL=$ relative distribution of income

*(e) capital-output ratio

$P/Y=P/(L^{1-a}P^{a})=(P/L)^{1-a}$ since $i=a(P/L)^{a-1}=aY/P$ or $Y/P=i/a$ then $\partial i/\partial P=-Y/P^{2}<0$ or there is an inverse relationship between i and P/Y.

d. Thus the parable tells us that, knowing only the quantity of putty and labor and the technology, we can find from the wage – interest rate frontier the corresponding wage and i rate that would rule under competitive conditions. The elasticity of the frontier at that pint gives the relative share of interest and profits. The distribution of income is therefore completely determined by technology and relative factor endowments. An increase in the quantity of one factor relative to the other lowers its price. The distribution of income varies accordingly, depending on the particular form of the technology, that is, depending on the elasticity of substitution. In this way, the analysis in corporates the argument that relative factor prices reflect relative scarcity of the different factors and the amount of which each factor gets from the national product is determined by technology and relative factor endowments.

1. while this modeling of the parable worked well in a one commodity world, problems start to arise in a multi-good world. Specifically, in a capitalist economy many different goods are capital goods, However if the above modeling is used to present neoclassical theory, the capital goods must be aggregated into single homogeneous magnitude. While economists, such as Hicks or Stigler, did not

think much of this point, Joan Robinson did since if the above modeling is to work then that aggregate amount of capital must be measured in a unit which in independent of distribution and prices, especially i. In this context, Joan Robinson made the following complaint: “The dominance in neoclassical economic teaching of the concept of a production function, in which the relative prices of the factors of production are exhibited as a function of the rates in which they are employed in a given state of technical knowledge, has had an enervating effect upon the development of the subject, for by concentrating upon the equation of the proportions of factors it has distracted attention from the more difficult but rewarding questions of the influences governing the supplies of the factors and of the causes and consequences of changes in technical knowledge. Moreover, the production function has been a powerful instrument of miseducation. The subtend of economic theory is taught to write O=f(L,C) where L is a quantity of labor, C a quantity of capital and O a rate of output of commodities. He is instructed to assume all workers alike, and to measure L in man-hours of labor; he is told something about the index=problem involved in choosing a unit of output; and he is hurried on the next question, in the hope that he will forget to ask in what units C is measured. Before ever he does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next. “

2. Joan Robinson’s query set off the capital controversy which listed into the 1970s and in the courses of controversy it was demonstrated that the simple parables cannot be extended beyond the one-good world, that there is no demand curve for capital, and that capital can not be seen as a factor of production on the same

footing with labor. Since the controversy is quite involved, it is not possible to give a detailed analysis of it; therefore I will only present the finished results so to speak.

<u>Cambridge Controversies in the Theory of Capital</u>

1. Let use consider a two-sector economy which is indecomposable and which has a maximum eigenvalue less than one. Such an economy can be represented by

   $(a_{11}p_1+a_{12}p_2)(1+i)+l_1w=p_1$

   $(a_{21}p_1+a_{22}p_2)(1+i)+l_2w=p_2$

(i) Obviously goods one and two are capital goods and at least one of them is also a consumption good.

(ii) $v=(I-A)^{-1}l$ where $v_{1,2}$ equals the total amount of labor embodied in each good, then we can state that $(v_1-l_1)/l_1$ is in general not equal to $(v_2-l_2)/l_2$. This implies that $(a_{11}p_1+a_{12}p_2)/l_1 \neq (a_{21}p_1+a_{22}p_2)/l_2$ for all i and w.

2. Let use now use this model to first determine whether the quantity of capital – now measured in value terms – can be determined in dependently of prices and distribution (this has already been discussed in the section on Wicksell).

a. Let us consider the following economy

   $(.487p_c+.21p_m)(1+i) + .104l_w=p_c$

   $(6p_c+.4p_m) (1+i) +.5l_w =p_m$

b. To derive the wage-interest rate frontier let us assume p_c=1; thus we get

   $W= (.1818-.7494i+.0688i^2) (.0729-.03155i)$

   And we know that dw/di<0 – i.e. the frontier slopes downward.

c. Now if we assume that the total output of corn is 575 and the total ouput of machines is 20 then total capital K will equal $K=400p_c+20p_m$. The questions we have to answer now is whether K is a quantity that is independent of prices, distribution and i.

(1) let us consider the following

$i=.25$, $w=0$, $p_c=1$, $p_m=15$, and $K=700$

$i=.20$, $w=.5365$, $p_c=1$, $p_m=.1436$, and $K=687.24$

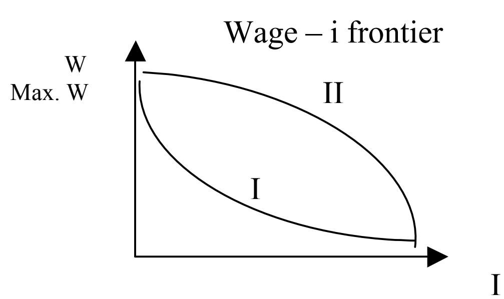
$i=.1$, $w=1.5522$, $p_c=1$, $p_m=13.17$, and $K=663.43$

$i=0$, $w=225$, $p_c=1$, $p_m=12.08$, and $K=641.65$

Thus we see that capital is not independent of prices or distribution.

(2) In this case the quantity of capital got smaller as the rate of interest fell – this is called a negative price Wicksell effect/

(3) There can also be positive price Wicksell effect in which the quantity of capital increases as i falls – but in either case, the quantity of capital is not independent of distribution or prices. The two Wicksell effects can be illustrated in the following manner



I. positive price Wicksell effect

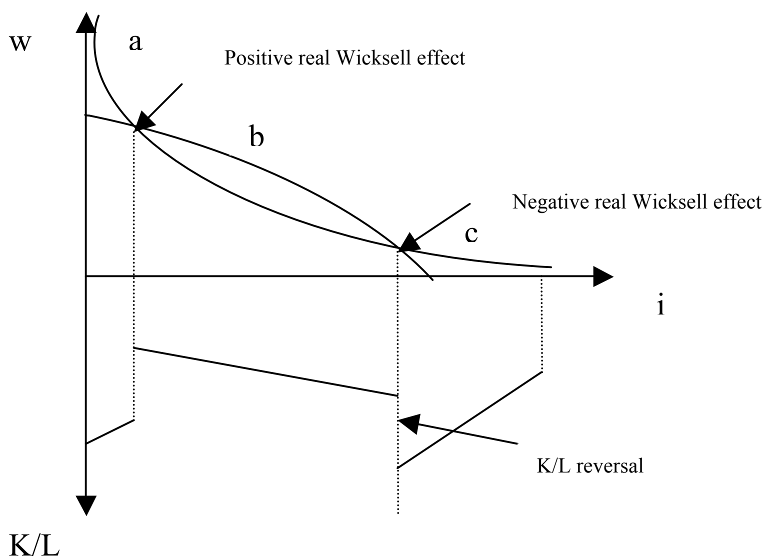
II.negative price Wicksell effect

(4) The implication of this result is that because K is a function of i, it cannot be used in the guise of a marginal product to ‘determine’i.

3. Let us now consider whether there is an inverse relationship between K and i, and between K/L and i.

a. First let us remember that each different technique represents a different K/L ratio with the larger the ratio the more capital intensive the technique.

b. Now let us first look at an economy with two different techniques



(1) A positive real Wicksell effect is one in which as i increases a technique is chosen at the switch point which has a lower K/L ratio.

(2) A negative real Wicksell effect is one in which as i increases a technique is chosen at the switch which has higher K/L ratio.

(3) In the above diagram we find the switching and reswitching of two techniques. The second switch results in a negative real Wicksell effect – thus implying that one cannot generally assume that there is an inverse relationship between i and K/L as postulated in the simple parallels models presented above. What is implied

in this result is that diminishing marginal product of capital associated with an decreasing i does not hold.

(4) The negative real Wicksell effect can also occur even if reswitching does not occur solely because the quantity of capital is not independent of prices and distribution.

(5) Now let us consider for the moment K and i, keeping L fixed. The negative real Wicksell effect and the associated K/L reversal clearly indicates that K and i are not in generally inversely related – i.e. there is no demand curve for capital. This point is of interest, therefore let us look at it some more.

(6) Let us consider the following set of techniques:

$M_cP_m(1+i)+L_cW=1$

$M_mP_m(1+i)+L_mW=1$

(a) Let $L_m=1$ $L_c=30+1/U^{11/10}+U^{11/5}-27e^{-2U}$

$M_m=(5+U^{11/10})/6+U^{11/10}$

$M_c=(27e^{-2U})/(6+U^{11/10})^2$ where e is the base of natural logarithms U=0, .25, .5, .75, 1, 1.25, 1.505

(b) the W-i frontier can be denoted as $W=[1-(5+U^{11/10})i] / [(5-U^{11/10})+ \{27e^{-2U}-(5+U^{11/20})^2\}i]$

(c) the data of the following i's

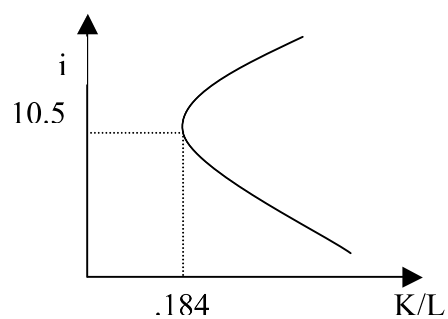
i	System in use U	W	K/L
0	0	.2	1.080
2.6	.25	.175	.635
4.1	.5	.169	.393

6.1	.75	.159	.257
8.3	1	.151	.184
10.5	1.25	.144	.148
12.9	1.505	.129	.179
14.4	1.25	.105	.379
15.1	1	.083	.552
15.9	.75	.061	.715
16.9	.5	.041	.85
17.5	.25	.026	.947
20.0	0	0	1

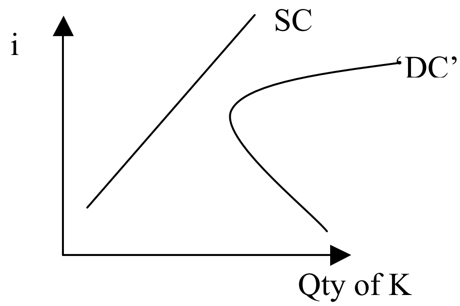
(d) W-i frontier and i-K/L

(i) Graph

(ii) i-K/L relationship



(iii) Because the demand curve for K assumes L is fixed, it takes on the same shape at the i-K/L curve. Thus it is easy to see that a demand curve for capital does not exist (this also implies that a demand curve for labor also does not exist). Moreover, because of the possible shape of the 'demand curve' it is possible that no market equilibrium will exist:



4. The implication of the capital controversy is far reaching. On the one hand, it showed that the Clarkian parables could not be extended beyond a one good world (expect in the special case of indirect-direct labor ratios being the same in al industries). Thus the distribution of income cannot be determined by technology and factor endowments and i and w do not reflect relative factor scarcity. moreover, it is not even possible to assume that equilibrium situations exist outside the simple Clarkian world. Finally, it should be noted that if the demand curve for capital does not exist, then the marginal efficiency schedules do not exist – i.e. most neoclassical macro-models are logically inconsistent.

In short, capital cannot be conceived as a factor of production on the same footing with labor – in fact capital in the Clarkian sense simply cannot exist.

CHAPTER 35

GENERAL EQUILIBRIUM: EXCHANGE AND PRODUCTION WITH NON-PRODUCED INPUTS

Methodology and Equilibrium

Quantity and Price Relations

V General Equilibrium – Theory of Production with Land and Labor

A. The Neoclassical theory of Resource Allocation: Quantity Relations

1. In the previous section, we only considered a pure exchange model – i.e. a model in which production did not take place. Now we are going to consider a particular kind of production model to see if the results obtained in the pure exchange model can be extended to a model of production. In particular we want to see whether production can be viewed as indirect exchange and whether prices reflect the relative scarcity of the factor inputs and of the exchange able inputs.
2. the model we will be working with has the following form:
    a. parameters of the model
        (1) the available quantities of various resources yielding factor services in production – in our case the factor inputs are land, T, and labor, L, and L and T represent the absolute amounts of these services available.
        (2) the technology of production specifying the ways in which factor services are transformable into final outputs – i.e. production functions are given

(3) the preferences of consuming agents for the various final products specified in the technology as well as for any directly consumable resources – utility functions are given and they contain only wheat and rice as the elements.

(4) the pattern of ownership among consuming agents of the given factors of production

b. constraints

(1) resource constraint

(a) available labor – services = labor – services used in rice production + labor – services used in wheat production

(b) available land – services = land – services used in rice production + land – services used in wheat production

(c) total output of wheat > 0 – $Y_w > 0$

total output of rice > 0 - $Y_r > 0$

(2) price constraints

(a) unit labor cost in rice production + unit land cost in rice production = price of a unit of rice

(b) unit of labor cost in wheat production + unit land cost in wheat production = price of unit of wheat

(c) price of labor >0 – $p_L > 0$

price of land >0 – $p_T > 0$

(3) models

(a) resource model

$a_{LR}\, Y_R + a_{LW}\, Y_W = L$

$a_{TR}\, Y_R + a_{TW}\, Y_W = T$

or

$$\begin{bmatrix} a_{LR} & a_{LW} \\ a_{TR} & a_{TW} \end{bmatrix} \begin{bmatrix} Y_r \\ Y_w \end{bmatrix} = \begin{bmatrix} L \\ T \end{bmatrix}$$

where

a_{LR} is the amount of labor-services needed to produce a unit of rice;

a_{LW} is the amount of labor-services needed to produce a unit of wheat;

a_{TR} is the amount of land-services needed to produce a unit of rice; and

a_{TW} is the amount of land-services needed to produce a unit of wheat

(b) price model

$a_{LR}\, p_L + a_{TR}\, p_T = p_R$

$a_{LW}\, p_L + a_{TW}\, p_T = p_W$

where

p_R is the price of rice

p_W is the price of wheat

(c) variables to determine

(1) allocation variables

(a) the allocation of factor services to different technical processes, and hence the outputs of various final commodities;

(b) the allocation of outputs to the various resource owners who are the model's consuming agents

(2) price variables

(a) factor price measuring the relative values of the different factor services;

(b) commodity prices measuring the relative values of various final outputs.

3. In this section we want to deal with the resource model and the determination of Y_R an Y_W and the allocation of L and T to the production of rice and wheat. Our analysis is based on the assumption that both land and labor are fully employed (hence the equal signs in the resource model).

a. before attempting a general solution to this problem, let us consider a specific numerical example:

(1) Let L = 120, T = 96 and A = $\begin{bmatrix} a_{LR} & a_{LW} \\ a_{TR} & a_{TW} \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 3 & 4 \end{bmatrix}$

therefore our resource constraints can be written as:

$120 = 6\ Y_R + 2\ Y_W$

$96 = 3Y_R + 4\ Y_W$

(2) Let us now consider each of the resource constraints separately

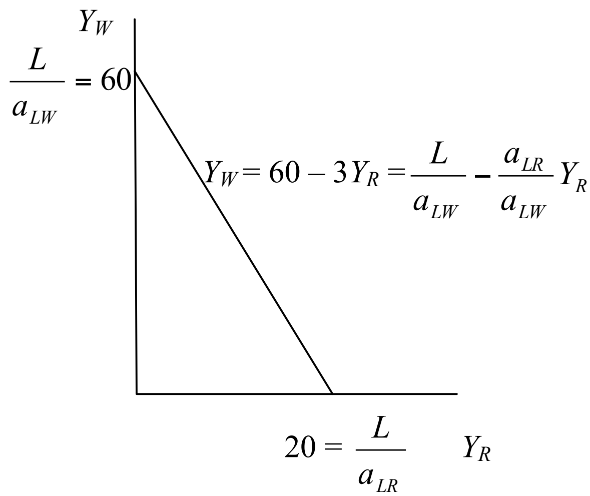
(a) $120 = 6\ Y_R + 2\ Y_W$

(i) if $Y_R = 0$, $Y_W = 60$

(ii) if $Y_W = 0$, $Y_R = 20$

(iii) $Y_W = 60 - 3\ Y_R$ (or $Y_w = \frac{L}{a_{LW}} - \frac{a_{LR}}{a_{LW} Y_R}$)

(iv) graphically we have:



(v) the line $Y_W = 60 - 3\ Y_R$ is a boundary of a set of points measuring nonnegative outputs of rice and wheat which do not violate the labor constraint. Points along the line are called limit points.

(vi) the slope of the line indicates the necessary reallocation of labor between the two sectors so as to maintain L fully utilized.

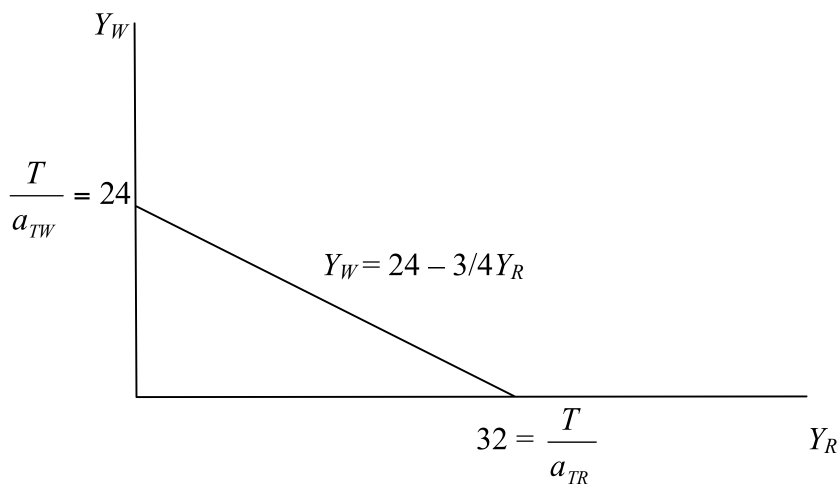
(b) $96 = 3Y_R + 4\ Y_W$

(i) if $Y_R = 0$, $Y_W = 24$

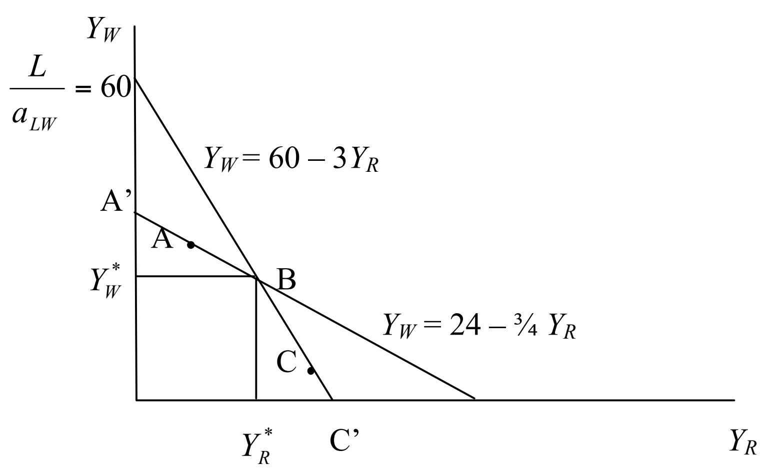
(ii) if $Y_W = 0$, $Y_R = 32$

(iii) $Y_W = 24 - \frac{3}{4}\ Y_R$ (or $Y_w = \frac{T}{a_{TW}} - \frac{a_{TR}}{a_{TW} Y_R}$)

(iv) graphically we have:



(3) considering the resource constraints together graphically we have



(a) the area encompasses by OA'BC' represents all the combinations of Y_R and Y_W which are feasible given L and T.

(b) point A represents an output combination which exhausts T but not L.

(c) point C represents output combination which exhausts L but not T.

(d) point B represents output combination which exhausts both L and T.

(e) now that Y_R^* and Y_W^* have been determined, it is now possible to determine the allocation of L and T to the production of rice and wheat:

(i) $a_{LW} Y_R^* = (6)(16) = 96_L$ the number of units of labor allocated for the production of rice

(ii) $a_{TR} Y_R = (3)(16) = 48_T$ the number of units of land allocated for the production of rice

(iii) $a_{LW} Y_W^* = (2)(12) = 24_L$ the number of units of labor allocated for the production of wheat

(iv) $a_{TW} Y_W^* = (4)(12) = 48_T$ the number of units of land allocated for the production of wheat

(v) $96_L + 24_L = 120_l$

$48_T + 48_T = 96_T$

b. a general solution to the problem of allocation can be derived through using Cramer's Rule

(1) resource model $\begin{bmatrix} a_{LR} & a_{LW} \\ a_{TR} & a_{TW} \end{bmatrix} \begin{bmatrix} Y_R \\ Y_W \end{bmatrix} = \begin{bmatrix} L \\ T \end{bmatrix} = AY = F$

(2) $$Y_R = \frac{\begin{vmatrix} L & a_{LW} \\ T & a_{TW} \end{vmatrix}}{|A|} = \frac{a_{TW} L - a_{LW} T}{a_{LR} a_{TW} - a_{LW} a_{TR}}$$

(3) $$Y_w = \frac{\begin{vmatrix} a_{LR} & L \\ a_{TR} & T \end{vmatrix}}{a_{LR} a_{TW} - a_{LW} a_{TR}} = \frac{a_{LR} T - a_{TR} L}{a_{LR} a_{TW} - a_{LW} a_{TR}}$$

c. condition for full employment – a necessary condition for the simultaneous full employment of both factors of production is that the two lines representing the boundaries of the resource constraints intersect at a point where both outputs are nonnegative. This requires that the ratio of labor to land in the economy as a whole, $\frac{L}{T}$, is neither greater than the ratio of labor to land in the production of the labor-intensive product (rice) nor less than he corresponding ratio in the production of the land-intensive product (wheat). Given $\frac{a_{LR}}{a_{TR}} > \frac{a_{LW}}{a_{TW}}$, it must be the case that: $\frac{a_{LR}}{a_{TR}} \geq \frac{L}{T} \geq \frac{a_{LW}}{a_{TW}}$.

Thus, as long as the aggregate factor ratio lies between the sectoral factor ratios, there is always some weighted average of the sectoral ratios (the weights being determined by the composition of output) such that the total supply of both factor services will be fully utilized.

(1) from our numerical example above

$$\frac{a_{LR}}{a_{TR}} = \frac{6}{3} = 2; \; \frac{L}{T} = \frac{120}{96} = 1.25; \; \frac{a_{LW}}{a_{TW}} = \frac{2}{4} = .5 \text{ so}$$

$2 > 1.25 > .5$

(2) working with the general model, the above results can be shown in the following manner:

(a) if $|A| > 0$ then $a_{LR}a_{TW} - a_{LW}a_{TR} > 0$ thus $\frac{a_{LR}}{a_{LW}} = \frac{a_{TR}}{a_{TW}}$

(b) if $|A| > 0$ and $Y_R > 0$ then $a_{TW}L - a_{LW}T > 0$ or $\frac{L}{T} > \frac{a_{LW}}{a_{TW}}$

(c) if $|A| > 0$ and $Y_W > 0$ then $a_{LR}T - a_{TR}L > 0$ or $\frac{L}{T} < \frac{a_{LR}}{a_{TR}}$

(d) thus if Y_R and $Y_w > 0$ and L and T are fully utilized then $\frac{a_{LR}}{a_{TR}} > \frac{L}{T} > \frac{a_{TR}}{a_{TW}}$ (from a,b,c above)

(e) if $|A| < 0$ then all signs become reversed, i.e. the labor-intensive sector becomes wheat and the land-intensive sector becomes rice.

4. opportunity cost
   a. it deals with the problem of how much of one good must be given up through a reallocation of resources to obtain more of the second good.
   b. working with our numerical example, we fid that the opportunity cost for the segment A'B is ¾ unit of wheat per unit of rice; for the segment BC' it is 3 units of wheat per unit of rice; and at point B the opportunity cost is both ¾ and 3 (a situation that always happens at corners). It should also be noted that the opportunity cost increases as more rice is produced in the place of wheat.
   c. it should be noted that opportunity cost reflects the existing technology and that it must always increase – i.e. if $|A| > 0$ and Y_R and $Y_w > 0$ then $\frac{a_{LR}}{a_{LW}} > \frac{a_{TR}}{a_{TW}}$
5. production as a linear process – from the above discussion we find that the transformation of inputs into outputs is a one-way flow from factors to final goods and services. This point will be discussed again under the section on capital.

B. The Neoclassical Theory of Resource Allocation: Dual Price Relations

1. prices as rates of exchange (or relative prices)
   a. the relevance of prices lies in their determining the exchange value of one good or factor service in terms of another. For this reason, only ratios of